PARAMETER ESTIMATION OF MULTIPLE PULSE TRAINS FOR ILLUMINATION SENSING

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ABSTRACT

We consider illumination sensing in a light emitting diode (LED) lighting system based on frequency division multiplexing (FDM). In the FDM scheme, LEDs render periodical illumination pulses at different frequencies with prescribed duty cycles. The purpose of illumination sensing is to identify the illumination contributions of different LEDs at a target location, within a limited response time. With the use of a photosensor at the target location, the problem of interest is to estimate the amplitude of the sensor signal component due to each LED. In this paper, we propose a successive estimation approach exploiting higher order harmonics of the sensor signal. We show that a larger number of LEDs can be supported using the proposed approach, as compared to our previous approach using only the fundamental frequency component, at the same estimation error.

Index Terms— Illumination sensing, LED lighting systems, frequency division multiplexing, successive estimation

1. INTRODUCTION

LED lighting systems are becoming prevalent, offering energy efficiency along with the ability to render colorful, dynamic and localized lighting effects [1]. This ability is due mainly to three features. First, LEDs are colored light sources by nature. Secondly, each LED can provide hundreds of different illumination levels through pulse width modulation (PWM) dimming [2]. Finally, an LED lighting system normally consists of a large number of spatially distributed LEDs with narrow light beams. Associated with these features is the challenge of controlling such lighting systems.

A solution to LED lighting control was presented in [3] using illumination sensing. Here, illumination pulses from different LEDs are modulated differently. A sensor is placed at the target location where a certain lighting effect is desired. The illumination contribution of each LED is in turn determined via sensor signal processing. A central controller can then determine the setting of the duty cycle for each individual LED to achieve the desired lighting effect. An alternate approach uses frequency division multiplexing (FDM) [4] and is attractive due to the low-cost driver circuits.

In this paper, we consider the FDM scheme, under which a typical optical signal received at the target location from the ith LED is illustrated in Fig. 1. The signal consists of a pulse train with frequency f_i, that is set differently for different LEDs. The frequencies $\{f_i\}$ are usually chosen between 2 and 4 kHz [4], which is determined by the requirement of no visible flicker and by optical properties of the LEDs. The amplitude of the pulse train is called the *illuminance* (in lux), denoted by a_i^o, and the duty cycle is denoted by p_i where $0 \leq p_i \leq 1$. The illumination contribution of the LED at the target location is given by the product $a_i^o p_i$. The duty cycle
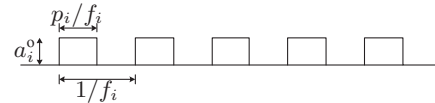

Fig. 1. Illustration of the optical signal due to the ith LED. The amplitude of the corresponding electrical pulse train is a_i.

p_i is known from the controller. The amplitude a_i^o, which is determined by the optical power of the ith LED and the free-space optical channel attenuation [3], is unknown. Hence, the key challenge for illumination sensing is to estimate a_i^o for each i to determine the illumination contribution of the ith LED.

The optical signals from all the LEDs superimpose at the target location. It is difficult and expensive to distinguish different LEDs optically. Therefore, the optical signal is converted into electrical by using a photosensor, such as a photodiode. The amplitude of the electrical sensor signal is denoted by a_i. The challenge of illumination sensing is in turn transformed into the estimation of a_i.

Based on the FDM scheme, an estimator was proposed in [4], where only the fundamental frequency component of the sensor signal was used. The basic idea was to estimate a_i through the spectrum of the sensor signal at f_i. Due to the limited response time for estimation (typically 0.1 s), there is induced interference between the sensor signal components from different LEDs. The frequency spacing between different f_i has, therefore, to be large enough to limit the induced interference. Given a fixed frequency range for the allocation of f_i, the number of LEDs that can be supported is thus limited. Moreover, the estimation performance deteriorates when the power of the fundamental frequency component is small, e.g. when p_i is close to 0 or 1. Finally, the estimation performance also suffers from frequency offsets in f_i. Due to these reasons, the number of LEDs that can be supported by this estimator is only about 100.

In many practical lighting applications, a larger number of LEDs need to be supported, e.g. [5]. To this end, we consider an estimation approach in this paper using higher order harmonics. This approach is attractive for two reasons. First, there is a larger frequency separation between the higher harmonics of two adjacent frequencies. Hence, it should be theoretically possible to pack a larger number of frequencies in the given frequency range. Secondly, we can utilize the power in the higher order harmonics in the sensor signal. This is especially meaningful when p_i is close to 0 or 1, in which case a larger portion of the signal power lies in higher order harmonics rather than the fundamental frequency component.

We note that previous literature on sinusoid estimation considered the case of parameter estimation with multiple sinusoids, e.g. [6, 7] and estimation using multiple harmonics from a single source, e.g. [8]. The problem setting and approach in this paper as such is fundamentally different from previous works.

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2. PROBLEM DESCRIPTION

The sensor signal at the output of the photosensor is given by

$$y(t) = \sum_{i=1}^{L} y_i(t) + v(t) = \sum_{i=1}^{L} a_i \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{(t+\theta_i)f_i - n}{p_i}\right) + v(t),$$

where L is the number of LEDs and is known from the controller. The signal $y_i(t)$ is the sensor signal component due to the ith LED, where $\text{rect}(t) = 1$ when $|t| \le 1/2$ and $\text{rect}(t) = 0$ elsewhere. Each $y_i(t)$ thus consists of a rectangular pulse train with an amplitude a_i, a frequency f_i and an initial phase shift θ_i. None of these three parameters is exactly known to the sensor. The amplitude a_i characterizes the illumination contribution of the ith LED, and is thus the primary parameter to be estimated.

The ideal frequency $\bar{f}_i$ is allocated between $f_{\min} = 2$ kHz and $f_{\max} = 4$ kHz, i.e. the considered bandwidth is $W = f_{\max} - f_{\min} = 2$ kHz. In [4], the $\{\bar{f}_i\}$ are allocated uniformly between $f_{\min}$ and $f_{\max}$ with a frequency spacing Δ_f between adjacent frequencies. In this paper, however, as will be explained later, we divide the frequency range $(f_{\min}, f_{\max})$ into M sub-ranges. The ideal frequencies $\{\bar{f}_i\}$ in each mth sub-range are still uniformly allocated with a frequency spacing $\Delta_{f,m}$, except that $\Delta_{f,m}$ decreases with the increase of m. Nevertheless, the value of $\bar{f}_i$ is always known to the sensor as the identifier for the ith LED. Due to clock inaccuracies, however, there is an unknown frequency offset, ϵ_i, between $\bar{f}_i$ and the actual frequency f_i, i.e. $\epsilon_i = f_i - \bar{f}_i$. The value of θ_i can be anywhere between $-\frac{1}{2f_i}$ and $\frac{1}{2f_i}$. To obtain accurate estimate of a_i, the values of f_i and θ_i also need to be estimated. The noise term $v(t)$, consisting of electronic and shot noise, is assumed to be additive white Gaussian noise with double-sided power spectrum density $\frac{N_0}{2}$. The signal-to-noise ratio (SNR) is defined as $\zeta_i = 10 \log_{10}(a_i^2/(N_0 W))$ [dB]. For a practical indoor environment [4], we have $\max_i(\zeta_i) \ge 90$.

To undertake the estimation of $\{a_i, f_i, \theta_i\}$, it is convenient to further write $y_i(t)$ into the form of harmonics, i.e.

$$y_i(t) = a_i \sum_{m=0}^{\infty} 2 b_{i,m} \cos(2\pi m f_i t + m\phi_i), \tag{1}$$

where $b_{i,m}$ is the magnitude of the mth harmonic. For a rectangular pulse with a known duty cycle p_i, we have known $b_{i,m}$ given by $b_{i,m} \approx \frac{\sin(\pi m p_i)}{\pi m}$. Note that, under practical protocols for PWM dimming control [4], we have $p_{\min} = 0.001 \le p_i \le p_{\max} = 0.973$. The phase term is $\phi_i = 2\pi f_i \theta_i$. The estimation of $\{a_i, f_i, \theta_i\}$ is then transformed to the estimation of $\{a_i, f_i, \phi_i\}$. In [4], only the term $m = 1$ in (1) is used for the estimation of a_i.

For the estimation of $\{a_i\}$, the key requirements concern speed and accuracy. The first requirement is that the response time $T \le 0.1$ s. The second requirement is that the estimation results should be accurate enough such that the estimation error is *invisible* to human eyes. More specifically, as long as the cost function [4]

$$\xi_i \triangleq 10 \log_{10}\left(|\hat{a}_i - a_i| p_i / (\sum_{m=1}^{L} a_m p_m)\right) \le -20\text{dB}, \tag{2}$$

where $\hat{a}_i$ denotes the estimated value for a_i, the estimation error is considered acceptable.

With the above requirements on speed and accuracy being satisfied, the key challenge is to design an estimator so as to accommodate more LEDs, compared to that in [4], by allowing a smaller spacing between adjacent frequencies. To do this, we shall exploit multiple harmonics.
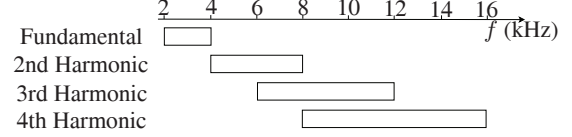

Fig. 2. Frequency Range for Different Harmonics.

3. SUCCESSIVE ESTIMATOR

3.1. Frequency Domain Perspective

We describe the proposed estimator from a frequency domain perspective. Due to the limited response time T, the sensor signal $y(t)$ is multiplied with a windowing function $g(t)$, whose support is T. In the Fourier domain, we have

$$Y(f) = \sum_{i=1}^{L} Y_i(f) + V(f), \tag{3}$$

where

$$Y_i(f) = \sum_{m=-\infty}^{\infty} Y_{i,m}(f) = \sum_{m=-\infty}^{\infty} a_i b_{i,m} e^{jm\phi_i} G(f - m f_i). \tag{4}$$

Here, $Y_i(f)$ is the Fourier transform of the windowed signal from the ith LED, i.e. $Y_i(f) = \int_{-\infty}^{\infty} y_i(t) g(t) e^{-j2\pi f t} \mathrm{d}t$, $Y_{i,m}(f)$ is the mth harmonic component in $Y_i(f)$ and $G(f)$ is the Fourier transform of $g(t)$. Without loss of generality, we let $G(0) = 1$. The Fourier transform of the windowed noise term is denoted by $V(f)$.

In order to utilize multiple harmonics for the estimation of $\{a_i, f_i, \phi_i\}$, we first look at the frequency ranges for different m, as illustrated in Fig. 2, where $m = 1$ to 4. It can be seen that the frequency ranges for different m overlap each other. This overlap indicates that there is mutual interference between different harmonics from different LEDs. Hence, it is not straightforward to use the mth harmonic and support m times more LEDs.

3.2. Successive Estimation Procedure

A closer inspection of Fig. 2 indicates that there is no frequency overlap in the frequency range between 4 kHz to 6 kHz in the second harmonic range. Hence, there is no interference from other harmonics. An estimator can thus be built based on this frequency range to obtain estimates, $\{\hat{a}_i, \hat{f}_i, \hat{\phi}_i\}$, for the LEDs with $4\,\text{kHz} < 2 f_i < 6\,\text{kHz}$. Let us name this estimator as the *Component Estimator*, as it is a component of the successive estimation process described later in this section.

Before describing such component estimators in further detail, the following frequency spacing criterion applies from [4]: *As long as the frequency spacing between two adjacent mth order harmonics is $2/T$, the estimates $\{\hat{a}_i, \hat{f}_i, \hat{\phi}_i\}$ are sufficiently accurate in the sense of (2), under response time T.* The above criterion applies under no interference from other harmonics. For $m = 2$, the corresponding spacing between two adjacent fundamental frequencies is only $1/T$. Hence, the frequencies $\{f_i\}$ can be more closely packed by a factor of two, in comparison to that in [4]. The above criterion however suggests that a larger number of LEDs can be supported using even higher order harmonics while maintaining the error estimation performance.

3.2.1. Successive Procedure

Now, instead of taking the frequency range between 4 kHz to 6 kHz, let us only consider the range from $2 f_{\min}$ to $\frac{4}{3} f_{\max}$, where we know
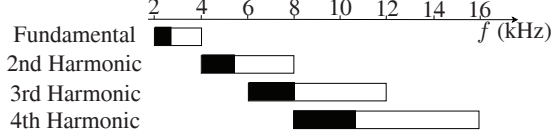
Fig. 3. Illustration of the successive estimator, where the dark regions correspond to the frequency ranges of the LEDs in Group 1.

group	W_m/W	harmonic	$\Delta_{f,m}$	L_m
1	$1/3$	2nd	$\frac{1}{T}$	$\frac{1}{3}WT$
$m < M$	$\frac{2}{(m+1)(m+2)}$	$(m+1)$th	$\frac{2}{(m+1)T}$	$\frac{1}{m+2}WT$
M	$\frac{2}{M+1}$	Mth	$\frac{2}{MT}$	$\frac{M}{M+1}WT$

Table 1. Frequency range and number of LEDs of each group.

$f_{\max} = 2f_{\min}$. Denote by *Group 1* all the LEDs with a second harmonic in this frequency range, or equivalently $f_{\min} < f_i < \frac{2}{3}f_{\max}$. The frequency ranges of different harmonics corresponding to Group 1 are illustrated in Fig. 3 by the dark blocks. Further denote the fundamental frequency range of Group 1 by W_1. Clearly, $W_1 = W/3$. From the above spacing criterion, we can support $L_1 = \frac{2W_1}{2/T} = \frac{1}{3}WT$ LEDs in the first group.

After estimating $\{\hat{a}_i, \hat{f}_i, \hat{\phi}_i\}$, for $i = 1$ to L_1, based on $\sum_{i=1}^{L_1} Y_{i,2}(f)$, we can also reconstruct $Y_{i,m}(f)$ for different m by $Y_{i,m}(f) \approx \hat{a}_i b_{i,m} e^{jm\hat{\phi}_i} G(f - m\hat{f}_i)$. Then if we subtract $\sum_m \sum_{i=1}^{L_1} Y_{i,m}$ from $Y(f)$, the overlap between the second and third harmonic ranges is removed, as seen from Fig. 3. Hence, using the component estimator in Section 3.3, we can estimate the parameters for the rest of the LEDs, named *Group 2*, based on their second harmonics, too. We can thus get $W_2 = \frac{2}{3}W$, and $L_2 = \frac{2W_2}{2/T} = W_2 T$. In this way, we can achieve $L = L_1 + L_2 = WT$ LEDs, which is twice as large as that in [4].

The above grouping and successive estimation procedure for two groups can be generalized to the case of $M > 2$ groups. When $m < M$, the range of $\{(m+1)f_i\}$ for the mth group is from $mf_{\max}$ to $(m+1)^2 f_{\max}/(m+2)$. The parameters of the LEDs are estimated based only on the $(m+1)$th harmonic signal. Hence, the equivalent frequency spacing between two adjacent frequencies in terms of $\{f_i\}$ is $\Delta_{f,m} = \frac{2/T}{m+1}$. The number of LEDs L_m can then be calculated as given in Table 1. Note that the last group simply takes the rest of the Mth harmonic range, so the property of the last group is slightly different from the previous groups.

From Table 1, the number of LEDs that can be supported is

$$L = WT\left(\sum_{m=1}^{M-1}\frac{1}{m+2} + \frac{M}{M+1}\right) = WT\left(\sum_{m=1}^{M}\frac{1}{m} - \frac{1}{2}\right), \quad (5)$$

which asymptotically scales logarithmically with M, considering the approximation $\sum_{m=1}^{M}\frac{1}{m} \approx \ln M + \gamma + O(M^{-1})$, where $\gamma \approx 0.577$. The numerical values of L are shown in Fig. 4.

Note that there is in practice an upper bound for M. This is because the frequencies $\{\bar{f}_i\}$ are allocated in a strict order, i.e. $\bar{f}_i < \bar{f}_{i+1}$, such that $\{\bar{f}_i\}$ are used as the identifiers of the LEDs. If, due to the frequency offsets, the order of the frequencies is changed, i.e. $f_i > f_{i+1}$ for some i, we will not be able to identify the LEDs successfully. To avoid this, assuming a 100 ppm clock inaccuracy, we should have a frequency spacing

$$\Delta_{f,M} = 2/(MT) > 2f_{\max} \times 100 \times 10^{-6} \approx 0.8 \text{ Hz}. \quad (6)$$
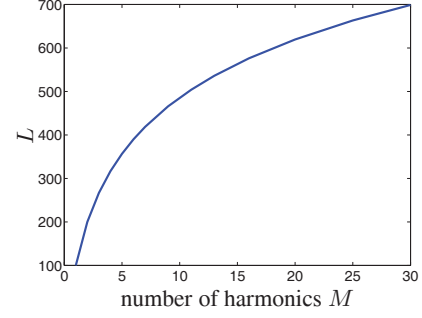
Thus, we have $M < 25$ when $T = 0.1$.

Fig. 4. Number of LEDs that can be supported.

3.3. The Component Parameter Estimator

In this section, we present a component parameter estimator that is used in the successive estimation procedure presented earlier in Section 3.2. This component parameter is applied once to each of the M groups. The aim of this estimator is to obtain an estimate $\{\hat{a}_i, \hat{f}_i, \hat{\phi}_i\}$ for each i in every group.

For Group $m-1$, the LEDs indexed are $i = \mathcal{L}_{m-2}+1$ to $\mathcal{L}_{m-1}$, where $\mathcal{L}_m = \sum_{k=1}^{m} L_k$. The input of such an estimator is denoted by $\mathcal{Y}_m(f)$, where $\mathcal{Y}_m(f) = Y(f) - \sum_{i=1}^{\mathcal{L}_{m-2}} Y_i(f)$. Moreover, we focus on the mth harmonic frequency range for this group, i.e. between $(m-1)f_{\max}$ and $m^2 f_{\max}/(m+1)$. Due to the successive estimation procedure, this is an overlap-free frequency range. Hence, we only need to consider

$$\mathcal{Y}_m(f) \approx \sum_{i=\mathcal{L}_{m-2}+1}^{\mathcal{L}_{m-1}} Y_{i,m}(f) = \sum_{i=\mathcal{L}_{m-2}+1}^{\mathcal{L}_{m-1}} a_i b_{i,m} e^{jm\phi_i} G(f - mf_i).$$

As shown in [4], we can design $G(f)$ such that $G(m\bar{f}_j - m\bar{f}_i) = 0$, for $i \neq j$, so that ideally we can use $\mathcal{Y}_m(m\bar{f}_i)$ to estimate a_i and $m\phi_i$. In practice, however, due to the existence of frequency offsets, we should in fact use $\mathcal{Y}_m(mf_i)$, instead of $\mathcal{Y}_m(m\bar{f}_i)$. Moreover, there may be induced interference from $Y_{j,m}(f)$ to the value $\mathcal{Y}_m(mf_i)$, since $mf_j - mf_i$ no longer lies on the zeros of $G(f)$. The resulting estimation error due to these effects of frequency offset is not tolerable, especially because of error propagation in the successive estimation procedure. Thus, due to the induced interference between $Y_{i,m}(f)$ and $Y_{j,m}(f)$, we subtract the interfering $Y_{j,m}(f)$ from $\mathcal{Y}_m(f)$ for the parameter estimation of the ith LED. Further, a more accurate estimation of f_i, instead of simply using $\bar{f}_i$, is desirable to arrive at better estimation of a_i and ϕ_i, too. To this end, we propose an iterative estimator as given in Algorithm 1.

The initial value of $\hat{f}_i$ for each i is set to be equal to the ideal frequency $\bar{f}_i$. The initial values of $\hat{a}_i$ is then obtained based on the magnitude of $\mathcal{Y}_m(\bar{f}_i)$. The phase of the mth harmonic is denoted by $\phi_{i,m} = \lfloor m\phi_i \rfloor$, where the operation $\lfloor \cdot \rfloor$ makes $-\pi \leq \phi_{i,m} \leq \pi$, such that $\phi_{i,m} - m\phi_i$ is a multiple of 2π. The initial estimate for the phase, denoted by $\hat{\phi}_{i,m}$, is obtained to be the phase of $\mathcal{Y}_m(\bar{f}_i)$. We can also then obtain the initial estimated mth harmonic component $\hat{Y}_{i,m}(f)$ for each i.

In each of the N_{iter} iterations, for the ith LED, we subtract the component $\hat{Y}_{j,m}(f)$ of the neighboring LEDs from $\mathcal{Y}_m(f)$, where $|j - i| < L_{\text{neighbor}}$, since these LEDs give the most significant interference to $\hat{Y}_{i,m}(f)$. In practice, it suffices to have $N_{\text{iter}} \leq 5$ and $L_{\text{neighbor}} \leq 10$. The result of this subtraction is considered to be an updated $\hat{Y}_{i,m}(f)$. Then $\hat{f}_i$ is obtained by locating the peak of $|\hat{Y}_{i,m}(f)|$ and $\hat{a}_i$ and $\hat{\phi}_i$ can in turn be obtained based on $\hat{Y}_{i,m}(\hat{f}_i)$.

Algorithm 1 An iterative parameter estimator

1: **for** $i = \mathcal{L}_{m-2} + 1$ to $\mathcal{L}_{m-1}$ **do**
2: $\quad \hat{f}_i \leftarrow \bar{f}_i, \hat{a}_i \leftarrow |\mathcal{Y}_m(m\hat{f}_i)/b_{i,m}|$
3: $\quad \hat{\phi}_{i,m} \leftarrow \text{angle}(\mathcal{Y}_m(m\hat{f}_i)/b_{i,m})$
4: $\quad \hat{Y}_{i,m}(f) \leftarrow \hat{a}_i b_{i,m} G(f - m\hat{f}_i) \exp(j\hat{\phi}_{i,m})$
5: **end for**
6: **for** N_{iter} iterations **do**
7: $\quad$ **for** $i = \mathcal{L}_{m-2} + 1$ to $\mathcal{L}_{m-1}$ **do**
8: $\qquad \hat{Y}_{i,m}(f) \leftarrow \mathcal{Y}_m(f) - \sum_j \hat{Y}_{j,m}(f)$, when $|j-i| < L_{\text{neighbor}}$
9: $\qquad \hat{f}_i \leftarrow \frac{1}{m} \arg_f \max |\hat{Y}_{i,m}(f)|$, when $|f - \bar{f}_i| < \frac{m\Delta_{f,m-1}}{2}$
10: $\qquad \hat{a}_i \leftarrow |\hat{Y}_{i,m}(m\hat{f}_i)/b_{i,m}|$
11: $\qquad \hat{\phi}_{i,m} \leftarrow \text{angle}(\hat{Y}_{i,m}(m\hat{f}_i)/b_{i,m})$
12: $\qquad \hat{Y}_{i,m}(f) \leftarrow \hat{a}_i b_{i,m} G(f - m\hat{f}_i) \exp(j\hat{\phi}_{i,m})$
13: $\quad$ **end for**
14: **end for**

In the result of Algorithm 1, besides $\hat{a}_i$ and $\hat{f}_i$, we can obtain $\hat{\phi}_{i,m}$, which could correspond to m possible $\hat{\phi}_i$, denoted by $\psi_{i,k}$ for $k = 1$ to m, such that $e^{jm\psi_{i,k}} = e^{jm\hat{\phi}_{i,m}}$. This is due to phase ambiguity. This phase ambiguity has to be resolved for the successive estimation procedure since $\hat{\phi}_{i,m+1}$ to $\hat{\phi}_{i,M}$ will be later used for the subtraction steps. To this end, we refer to the $(m-1)$th harmonic range of the LEDs in the $(m-1)$th group. Specifically, we consider

$$\mathcal{Y}_{m-1}(f) \approx \sum_{i=\mathcal{L}_{m-2}+1}^{\mathcal{L}_{m-1}} a_i b_{i,m-1} e^{j(m-1)\phi_i} G(f-(m-1)f_i). \quad (7)$$

Theoretically, for every possible $\psi_{i,k}$, we can reconstruct $\hat{Y}_{i,m-1}(f)$. Among these $\{\psi_{i,k}\}$, the one that makes $\hat{Y}_{i,m-1}(f)$ match best with the $\mathcal{Y}_{m-1}(f)$ at $f = (m-1)\hat{f}_i$ is taken as $\hat{\phi}_i$. In practice, due to the induced interference from the neighboring LEDs, we also consider the $(i-1)$th and $(i+1)$th LEDs jointly with the ith LED. Specifically, there are, in total m^3 different combinations of $\{\psi_{i-1,k_1}, \psi_{i,k_2}, \psi_{i+1,k_3}\}$, where $k_1, k_2, k_3 \in \{1, 2, \ldots, m\}$.

Let $\mathbf{k} = [k_1, k_2, k_3]$. For every possible $\mathbf{k}$, we can regenerate $\hat{Y}_{l,m-1}(f)$, for $l = \{i-1, i, i+1\}$. The optimal $\mathbf{k}^* = [k_1^*, k_2^*, k_3^*]$ is given by

$$\mathbf{k}^* = \arg_{\mathbf{k}} \min \left| \mathcal{Y}_{m-1}((m-1)\hat{f}_i) - \sum_{l=i-1}^{i+1} \hat{Y}_{l,m-1}((m-1)\hat{f}_i) \right|,$$

and the final estimate is obtained as $\hat{\phi}_i = \psi_{i,k_2^*}$. The phase ambiguity is as such resolved.

Finally, note that the mth and $(m-1)$th harmonics of every LED in the $(m-1)$th group should exist for the above described component estimator to work effectively. This gives a minor constraint on the duty cycles. In particular, it is required that each p_i in the $(m-1)$th group has to satisfy that both $|b_{i,m}|$ and $|b_{i,m-1}|$ are larger than δ, where δ is small (e.g. $\delta = 0.001$ in chosen in the numerical results presented next).

4. NUMERICAL RESULTS

Due to a large number of combinations of $\{p_i, a_i, f_i, \phi_i\}$, it is not possible to show the performance for each parameter set. By way of illustration, we show the case when $p_i = p_{\min}$ for every i, when there is significant signal power in the higher harmonics. Further, we take $a_i = a_k$ for every $i \neq k$, such that every LED receives significant induced interference from other LEDs. The frequency offsets ϵ_i and
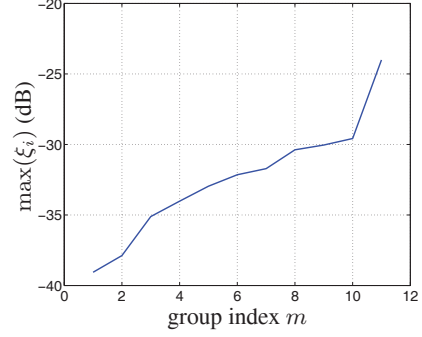

Fig. 5. Estimation performance of each group when $M = 11$, for $L = 500$.

ϕ_i are generated randomly, based on uniform distributions, according to 100 ppm clock inaccuracy and $-\pi \leq \phi_i \leq \pi$, respectively.

Results are shown for $M = 11$, $L = 500$ and $\zeta_i = 90$ dB. Specifically, the estimation error $\max_i(\xi_i)$ for the LEDs in each group is depicted in Fig. 5. Here, the simulations were performed 100 times and the worst case was selected for each $\max_i(\xi_i)$ among the 100 simulation instances. It can be seen that there tends to be a larger error in the later groups than the earlier groups, attributed to error propagation and larger frequency offsets. Nevertheless, the estimation error is well below -20 dB for every LED, as required.

5. CONCLUSIONS

We considered the use of multiple harmonics for parameter estimation in illumination sensing for FDM based LED lighting systems. A successive estimation approach was presented. With this approach, the number of LEDs that can be supported scales logarithmically with the number of harmonics considered. Simulation results indicate that we can support at least five times the number of LEDs in comparison to the method previously described in [4].

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