Impact and Digital Estimation of Nonlinear Impairments in Analog Baseband Processing Stages

Admar M.J.M. Schoonen*, Tim C.W. Schenk°, Peter G.M. Baltus*

* Eindhoven University of Technology, Department of Electrical Engineering, Eindhoven, The Netherlands, a.m.j.m.schoonen@tue.nl, p.g.m.baltus@tue.nl

° Philips Research, Distributed Sensor Systems department, Eindhoven, The Netherlands, tim.schenk@philips.com

***Abstract*— The digital compensation of RF imperfections is promising, but requires accurate models for the influence of the impairments on the baseband signals. To this end, this paper proposes a model for imperfections occurring in the popular direct conversion receiver architecture. It extends the previously applied IQ imbalance models by also taking DC offsets and nonlinearities from the output stage of the mixer up to the input stage of the ADC into account. Moreover, the impact of this more extended impairment model on the received baseband signals is derived. Finally, an estimation algorithm is presented to estimate the parameters of this model for digital compensation of the influence of these imperfections. Numerical results show that this estimation approach can achieve accurate estimates of the impairment parameters.**

I. Introduction

The digital compensation of nonlinear imperfections induced by radio frequency (RF) front-ends is foreseen to be a key technology for next-generation high-rate wireless systems [1]. In this approach a certain level of imperfection of the RF analog circuits is accepted and digital signal processing algorithms are applied to suppress their impact on system performance. These estimation and compensation algorithms can be applied offline during a calibration period or realtime during normal transceiver operation. For such techniques to be effective, however, a good understanding and modeling of the imperfections is crucial. The accurate modeling of these imperfections for all stages between the output of the mixer and the input of the ADCs is addressed in this paper.

The key performance properties of the analog and RF components in the design of a wireless communication system are, amongst others, gain, power, noise and linearity. Traditionally, linearity is characterized using the IP3 and 1-dB compression points [2]. These performance measures, however, do not unambiguously relate to a mathematical expression for the distorted signal. Therefore, approaches have been applied to model the nonlinear signal distortion due to individual transistors using a Taylor series approach and to extend this model to components such as amplifiers and mixers, see e.g. [3] and [4]. These models are limited to device or at most circuit level and, hence, are not directly applicable at the considered system level.

For system level modeling of nonlinear components with memory, the use of Volterra series has been proposed, see e.g. [5] and [6]. The complexity of these models is, however, relatively high which makes their applicability for our approach limited. Several attempts have been made to reduce the complexity of these models, see e.g. [7] and [8], but a simple and compact system model that expresses the influence of nonlinearities in mixers, baseband filters and amplifiers has not yet been presented to the best of the authors' knowledge.

This motivates the work presented in this paper, which proposes a system level impairment model for the effects of imperfect mixers, filters and amplifiers in the baseband branches of direct conversion receivers. The nonlinearity is modeled from the output of the mixer up to the input of the ADC, since in most communication systems these are the last stages that provide gain. Therefore, these stages have to handle the largest signals and are, thus, the most likely source of distortions. For simplicity, our model assumes that the mixer and the components after the mixer do not exhibit significant memory effects. This is validated by the trend towards very wide-band systems which perform part of the channel selection in digital domain. In such systems, the signal of interest and its distortion components are hardly affected by memory effects. Next to the nonlinearities, the model also takes into account the other typical baseband imperfections, i.e., mismatch in amplitude and gain between the mixers in the in-phase (I) and quadrature (Q) branches and DC offsets of individual components. Finally, a simple digital estimation algorithm is proposed to estimate all parameters of this model to be applied for system calibration and digital compensation.

II. System Level Baseband Impairment Model

A. Derivation

A simple, but commonly applied, system model for a direct down conversion mixer considers only the very essential behavior of the mixer, i.e., converting the received signal from RF to baseband. Assuming a Gilbert cell mixer, this behavior can be modeled by two branches, one which multiplies the received RF signal with $\cos(\omega_c t)$ and one which multiplies this signal with $-\sin(\omega_c t)$ and filters to suppress the high frequency components, as is illustrated in Fig. 1.
This yields the following baseband signals

$$\begin{aligned} y_I(t) &= r(t)\cos(\omega_c t), \\ y_Q(t) &= -r(t)\sin(\omega_c t), \end{aligned} \quad (1)$$

where ω_c is the carrier (RF) frequency, $r(t)$ is the received RF signal and $y_I(t)$ and $y_Q(t)$ are the in-phase and quadrature signal after down conversion, respectively.

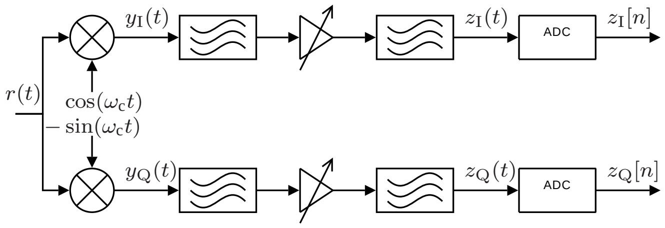

Fig. 1. System model for a non-impaired down conversion receiver

This model can be extended to take account for gain and phase offsets for the mixers in the I and Q branch, resulting in a system level model with IQ mismatch:

$$\begin{aligned} y_\mathrm{I}(t) &= \beta_\mathrm{I} r(t)\cos(\omega_\mathrm{c}t+\phi_\mathrm{I}), \\ y_\mathrm{Q}(t) &= -\beta_\mathrm{Q} r(t)\sin(\omega_\mathrm{c}t+\phi_\mathrm{Q}), \end{aligned} \tag{2}$$

where β_I and β_Q model the gain mismatch and ϕ_I and ϕ_Q model the phase mismatch. The modeling of IQ mismatch is more and more applied in literature [9].

The other imperfections at the output of the mixer and other baseband processing elements are, however, generally neglected and not taken into account in system models presented in previous literature. If we here do consider the baseband processing components to be slightly nonlinear, the resulting output signals can be approximated using an Mth order Taylor series, which is given by

$$\begin{aligned} y_\mathrm{I}(t) &= \sum_{m=0}^{M-1} \beta_{\mathrm{I},m} r^m(t)\cos^m(\omega_\mathrm{c}t+\phi_{\mathrm{I},m}), \\ y_\mathrm{Q}(t) &= -\sum_{m=0}^{M-1} \beta_{\mathrm{Q},m} r^m(t)\sin^m(\omega_\mathrm{c}t+\phi_{\mathrm{Q},m}). \end{aligned} \tag{3}$$

Since the most dominant components are those up to third order, expanding this for a third order approximation for the I branch signal yields

$$\begin{aligned} y_\mathrm{I}(t) = \beta_{\mathrm{I},0} &+ \beta_{\mathrm{I},1} r(t)\cos(\omega_\mathrm{c}t+\phi_{\mathrm{I},1}) \\ &+ \beta_{\mathrm{I},2} r^2(t)\cos^2(\omega_\mathrm{c}t+\phi_{\mathrm{I},2}) \\ &+ \beta_{\mathrm{I},3} r^3(t)\cos^3(\omega_\mathrm{c}t+\phi_{\mathrm{I},3}). \end{aligned} \tag{4}$$

We note that the third order approximation for the Q branch is similar to (4). In these expressions $\beta_{\mathrm{I},0}$ and $\beta_{\mathrm{Q},0}$ model the DC offset of the I and Q branch, respectively. Since the second and third order distortions result in a.o. signal components at two and three times the carrier frequency, the phase shifts at those frequencies, i.e. $\phi_{\mathrm{X},2}$ and $\phi_{\mathrm{X},3}$, are modeled independent of $\phi_{\mathrm{X},1}$. Note that all β parameters are typically close to 0, except for $\beta_{\mathrm{X},1}$, which is close to 1. Furthermore, all ϕ parameters can, in principle, have any value between $-\pi$ and π, but typically $\phi_{\mathrm{X},1}$ is close to 0. For a perfectly linear system, $\beta_{\mathrm{X},1}=1$ and all other parameters in (4) equal 0.

Combining (4) for the I and Q branch by $y(t) = y_\mathrm{I}(t) + iy_\mathrm{Q}(t)$ results in

$$\begin{aligned} y(t) = \mu_{0,0,a} &+ 2r^2(t)\mu_{2,0,a} \\ &+ 2r(t)\left(e^{i\omega_\mathrm{c}t}\mu_{1,1,a} + e^{-i\omega_\mathrm{c}t}\mu_{1,1,b}\right) \\ &+ \frac{8}{3}r^3(t)\left(e^{i\omega_\mathrm{c}t}\mu_{3,1,a} + e^{-i\omega_\mathrm{c}t}\mu_{3,1,b}\right) \\ &+ 4r^2(t)\left(e^{i2\omega_\mathrm{c}t}\mu_{2,2,a} + e^{-i2\omega_\mathrm{c}t}\mu_{2,2,b}\right) \\ &+ 8r^3(t)\left(e^{i3\omega_\mathrm{c}t}\mu_{3,3,a} + e^{-i3\omega_\mathrm{c}t}\mu_{3,3,b}\right), \end{aligned} \tag{5}$$

where the reader is referred to the Appendix for the definition of the μ parameters used in this model and the relation between these parameters and the parameters of the model in (3).

Subsequently, we want to relate the expression of the distorted received baseband signal to the transmitted baseband signal, since this can reveal the influence of the imperfections on data detection. To this end, we can write the received RF signal $r(t)$ as

$$r(t) = \frac{1}{2}\left(e^{i\omega_\mathrm{c}t}(x(t)*h(t)) + e^{-i\omega_\mathrm{c}t}(x^*(t)*h^*(t))\right), \tag{6}$$

where the baseband transmit signal is denoted as $x(t)$ and $h(t)$ is the baseband equivalent channel impulse response. When (6) is substituted in (5) and if the received signal $y(t)$ is fed through an ideal low pass filter with a cut-off frequency of ω_c, the resulting signal $z(t)$ is given by

$$z(t) = \mathrm{LPF}\{y(t)\} = \mathbf{m}^\mathrm{T}\mathbf{x}(t) \tag{7}$$

with

$$\begin{aligned} \mathbf{m} = [\mu_{0,0,a} \quad \mu_{2,0,a} \quad \mu_{1,1,a} \quad \mu_{1,1,b} \quad \mu_{3,1,a} \quad \ldots \\ \mu_{3,1,b} \quad \mu_{2,2,a} \quad \mu_{2,2,b} \quad \mu_{3,3,a} \quad \mu_{3,3,b}]^\mathrm{T} \end{aligned} \tag{8}$$

and

$$\begin{aligned} \mathbf{x}(t) = [1 \quad |x_h(t)|^2 \quad x_h^*(t) \quad x_h(t) \quad x_h^*(t)|x_h(t)|^2 \quad \ldots \\ x_h(t)|x_h(t)|^2 \quad x_h^{*2}(t) \quad x_h^2(t) \quad x_h^{*3}(t) \quad x_h^3(t)]^\mathrm{T}, \end{aligned} \tag{9}$$

where $x_h(t) = (x*h)(t)$, a^* denotes the conjugate of a, and $(a*b)(t)$ is the convolution of signals $a(t)$ and $b(t)$.

The result in (7) shows the influence of the impairments in the analog baseband processing stages, including DC offset, IQ imbalance and second and third order output nonlinearities, on the relation between the transmitted and received baseband signal. This system model can be used for the development of digital compensation algorithms.

B. Link to traditional performance measures

It is subsequently useful to map the parameters of the model introduced above to the performance measures used generally to characterize the distortion of RF and analog circuits. The first one is input IP3 (IIP3), for which there are various definitions, as shown in [10, Appendix H]. Since IIP3 typically is related to power and has unit dBm, the traditional definitions of IP3 are not useful for a system level model without units. Therefore, we define IIP3 as the strength of the input signal where the strength of the third order output compontents is equal to the strength of the linear output signal, which, according to the historical dual tone IP3 definition, is given by

$$\mathrm{IIP3} = \frac{2|\mu_{1,1,b}|}{3(|\mu_{3,1,a}| + |\mu_{3,1,b}| + |\mu_{3,3,a}| + |\mu_{3,3,b}|)}. \tag{10}$$

Similarly, we define the relationship between the distortion model and the input IP2 as

$$\mathrm{IIP2} = \frac{|\mu_{1,1,b}|^2}{(|\mu_{2,0,a}| + |\mu_{2,2,a}| + |\mu_{2,2,b}|)^2}. \tag{11}$$

Furthermore, the linear signal strength is defined as $|\mu_{1,1,b}|^2$, the strength of the DC offset is $|\mu_{0,0,a}|^2$, and the amount of IQ leakage is defined as $|\mu_{1,1,a}|^2$.

III. Parameter Estimation

The first, and most crucial, step in the digital compensation of RF imperfections is the estimation of the impairment parameters. Therefore we will focus on this step here. It is convenient to enable the estimation of the parameters of the impairment model in (7) by the transmission of pilot symbols. If we consider the baseband equivalent system, as schematically depicted in Fig. 2, the pilot sequence $p[n]$ of length N is generated by the transmitter and consists of an in-phase part $p_\mathrm{I}[n]$ and a quadrature part $p_\mathrm{Q}[n]$. These sequences are converted to an analog representation and low pass filtered, resulting in $p_\mathrm{I}(t)$ and $p_\mathrm{Q}(t)$. The analog pilot signal is given by $p(t) = p_\mathrm{I}(t) + ip_\mathrm{Q}(t)$.

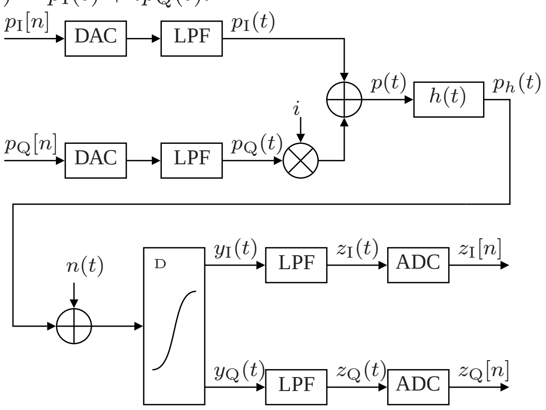

Fig. 2. Baseband equivalent system model for the transmit and receive chain.

The pilot sequence is transmitted and passes through a channel with impulse response $h(t)$. The resulting signal $p_h(t)$ is corrupted by additive white Gaussian noise and the resulting signal is distorted according to our model by the block indicated with D, split in in-phase and quadrature parts and low pass filtered, resulting in signals $z_\mathrm{I}(t)$ and $z_\mathrm{Q}(t)$. After sampling, we obtain the discrete signals $z_\mathrm{I}[n]$ and $z_\mathrm{Q}[n]$.

If we neglect the noise signal $n(t)$ for the moment, the receiver can estimate the parameters of the nonlinear model by combining the samples $z_\mathrm{I}[n]$ and $z_\mathrm{Q}[n]$ in signal $z[n]$ using $z[n] = z_\mathrm{I}[n] + iz_\mathrm{Q}[n]$. Assuming the channel impulse response $h(t)$ is known at the receiver [1], the receiver can create a modified Vandermonde matrix of the pilot signal $p_h[n]$ according to

$$\mathbf{P} = [\mathbf{p}[1] \quad \mathbf{p}[2] \quad \dots \quad \mathbf{p}[N]] \tag{12}$$

with

$$\mathbf{p}[n] = \big[1 \quad |p_h[n]|^2 \quad p_h^*[n] \quad p_h[n] \quad p_h^*[n]|p_h[n]|^2 \dots \\ p_h[n]|p_h[n]|^2 \quad p_h^{*2}[n] \quad p_h^2[n] \quad p_h^{*3}[n] \quad p_h^3[n]\big]^T \tag{13}$$

and $p_h[n] = (p * h)[n]$. The received signal $z[n]$ for $n = 1 \dots N$ can now be expressed as column vector $\mathbf{z}$ by

$$\mathbf{z}^\mathrm{T} = \mathbf{m}^\mathrm{T}\mathbf{P}. \tag{14}$$

Consequently we can estimate the parameters of the impairment model for analog baseband processing stages in vector $\mathbf{m}$ using Least Squares estimation. This results in the estimated parameter vector

$$\widetilde{\mathbf{m}} = \{\mathbf{z}^\mathrm{T}\mathbf{P}^\dagger\}^\mathrm{T} \tag{15}$$

where $\mathbf{P}^\dagger$ is the pseudo inverse of $\mathbf{P}$, defined as $\mathbf{P}^\dagger = \mathbf{P}^\mathrm{H}\left(\mathbf{P}\mathbf{P}^\mathrm{H}\right)^{-1}$.

Note that the proposed estimation approach does not exploit the dependencies between the different parameters given in (17)-(19). Improved performance could thus be achieved by exploiting this. Section IV will, however, show that even without exploiting these dependencies, the estimator achieves a good performance.

Compensation for the influence of the imperfect baseband components can subsequently be performed by applying digital postdistortion techniques, such as based on polynomial interpolation techniques, as for instance described in [1, Section 6.5].

IV. Numerical Results

Monte Carlo simulations were conducted to evaluate the performance of the estimator proposed in the previous section. For this evaluation an additive white Gaussian noise (AWGN) channel was simulated. This scenario can correspond both to calibration in the receiver using a locally generated pilot signal as to transmission of a preamble over a non-dispersive channel for estimation. A pseudo-random Gaussian distributed pilot sequence of length 100 was used. The parameters of the impairment model for the simulations are given in Table I.

TABLE I

Simulation parameters

$\mu_{0,0,a}$	$1.7 - 1.3i$
$\mu_{1,1,a}$	$0.08 - .12i$
$\mu_{1,1,b}$	$1.88 + 0.09i$
$\mu_{2,2,a}$	$0.025 + 0.01i$
$\mu_{2,2,b}$	$0.03 - 0.03i$
$\mu_{3,1,a}$	$0.14 + 0.20i$
$\mu_{3,1,b}$	$0.1 - 0.150i$
IIP2	31.0 dB
IIP3 2-tone historical	6.75 dB
Signal strength	-6.55 dB
DC power	6.61 dB
IQ leakage	-28.9 dB

[1]The channel impulse response can be known if the receiver is in calibration mode where transmit chain is fed via a known path to the receive chain, or alternatively, if the system uses a specially designed preamble with very low PAPR which will not significantly be distorted by the nonlinearities.

Figure 3 depicts the mean squared errors (MSEs) of the estimated parameters of the model versus signal-to-noise ratio (SNR). We can see that the MSEs of all 10 parameters decrease linearly with SNR on a log-log scale. At an SNR of 20 dB, the MSE of all parameters is approximately $2 \cdot 10^{-6}$ or lower.

Furthermore, Fig. 4 shows the MSE in the estimation of the distorted signal using the estimated parameters, i.e., the error in the estimate of $\mathbf{z}$ found by replacing $\mathbf{m}$ by $\widetilde{\mathbf{m}}$ in (14). This measure is related to the performance which can be achieved in postdistortion using the estimated parameters. The MSE also decreases linearly with SNR on a log-log scale and no performance floor is observed.

From these results we can conclude that the estimator, as proposed in Section III, performs very well, despite the fact that it does not exploit all information of the proposed model.

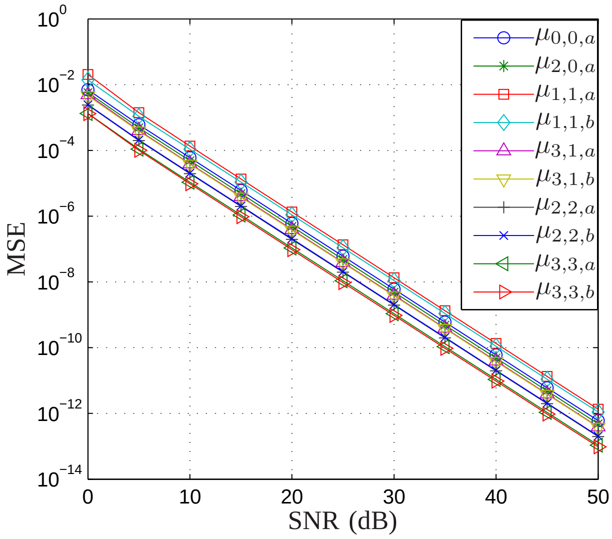

Fig. 3. MSE in estimated baseband impairment model parameters.

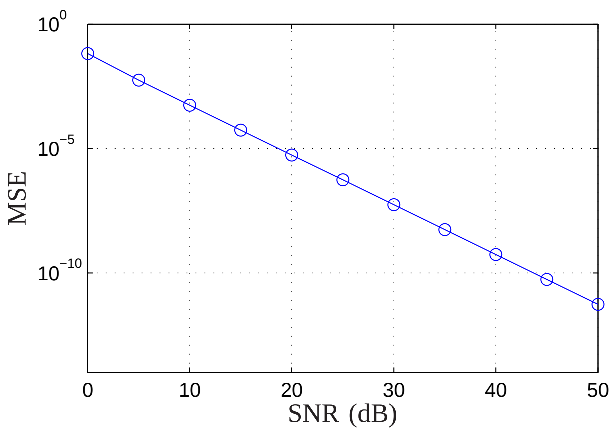

Fig. 4. MSE in estimated distorted signal.

V. Conclusions and Discussion

To facilitate progress in the field of digital compensation of RF impairments, this paper developed a new system level model for imperfect direct down conversion receivers. The model captures the main impairments in the analog baseband processing stages, i.e. DC offsets, IQ imbalance and 2nd and 3rd order nonlinearities. The model was used to derive a low-complexity estimation algorithm, which was shown to be able to accurately estimate the parameters of the model.

Future research could include the verification of the suitability of the proposed model by performing CAD simulations and/or RF system measurements. Also, the model and the algorithm could be extended for other RF imperfections. The algorithm could be improved by exploiting the dependencies between the parameters, as defined in (17)-(19). Finally, a specific digital baseband algorithm could be designed to remove the distortion from the received signal, which will reveal the resulting system performance.

Appendix

The μ parameters in (5) are defined as

$$\begin{aligned}
\mu_{0,0,a} &= \beta_{\mathrm{I},0} - i\beta_{\mathrm{Q},0} \\
\mu_{2,0,a} &= (1/4)(\beta_{\mathrm{I},2} - i\beta_{\mathrm{Q},2}) \\
\mu_{1,1,a} &= (1/4)(\beta_{\mathrm{I},1}e^{i\phi_{\mathrm{I},1}} - \beta_{\mathrm{Q},1}e^{i\phi_{\mathrm{Q},1}}) \\
\mu_{1,1,b} &= (1/4)(\beta_{\mathrm{I},1}e^{-i\phi_{\mathrm{I},1}} + \beta_{\mathrm{Q},1}e^{-i\phi_{\mathrm{Q},1}}) \\
\mu_{3,1,a} &= (9/64)(\beta_{\mathrm{I},3}e^{i\phi_{\mathrm{I},3}} - \beta_{\mathrm{Q},3}e^{i\phi_{\mathrm{Q},3}}) \\
\mu_{3,1,b} &= (9/64)(\beta_{\mathrm{I},3}e^{-i\phi_{\mathrm{I},3}} + \beta_{\mathrm{Q},3}e^{-i\phi_{\mathrm{Q},3}}) \\
\mu_{2,2,a} &= (1/16)(\beta_{\mathrm{I},2}e^{i2\phi_{\mathrm{I},2}} + i\beta_{\mathrm{Q},2}e^{i2\phi_{\mathrm{Q},2}}) \\
\mu_{2,2,b} &= (1/16)(\beta_{\mathrm{I},2}e^{-i2\phi_{\mathrm{I},2}} + i\beta_{\mathrm{Q},2}e^{-i2\phi_{\mathrm{Q},2}}) \\
\mu_{3,3,a} &= (1/64)(\beta_{\mathrm{I},3}e^{i3\phi_{\mathrm{I},3}} + \beta_{\mathrm{Q},3}e^{i3\phi_{\mathrm{Q},3}}) \\
\mu_{3,3,b} &= (1/64)(\beta_{\mathrm{I},3}e^{-i3\phi_{\mathrm{I},3}} - \beta_{\mathrm{Q},3}e^{-i3\phi_{\mathrm{Q},3}})
\end{aligned} \quad (16)$$

The relationships between the μ parameters are given by:

$$\mu_{2,0,a} = 2|\mu_{2,2,a} + \mu_{2,2,b}^*| - i2|\mu_{2,2,a} - \mu_{2,2,b}^*| \quad (17)$$

$$\mu_{3,3,a} = \frac{1}{18}\frac{(\mu_{3,1,a} + \mu_{3,1,b}^*)^2}{\mu_{3,1,a}^* + \mu_{3,1,b}} + \frac{1}{18}\frac{(\mu_{3,1,b}^* - \mu_{3,1,a})^2}{\mu_{3,1,b} - \mu_{3,1,a}^*} \quad (18)$$

$$\mu_{3,3,b} = \frac{1}{18}\frac{(\mu_{3,1,a}^* + \mu_{3,1,b})^2}{\mu_{3,1,a} + \mu_{3,1,b}^*} - \frac{1}{18}\frac{(\mu_{3,1,b} - \mu_{3,1,a}^*)^2}{\mu_{3,1,b}^* - \mu_{3,1,a}} \quad (19)$$

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