

On the Influence of Phase Noise Induced ICI in MIMO OFDM Systems

Tim C. W. Schenk, *Student Member, IEEE*, Xiao-Jiao Tao, *Member, IEEE*,
Peter F. M. Smulders, *Senior Member, IEEE*, and Erik R. Fledderus

Abstract—The influence of transmitter and receiver phase noise (PN) on the performance of a multiple-input multiple-output orthogonal frequency division multiplexing (MIMO OFDM) based communication system is analyzed. It is shown that in the case of frequency flat Rayleigh fading, the influence of receiver and transmitter PN is equal. In the case of independent Rayleigh fading, however, the impact of the receiver PN is shown to depend on the ratio between the number of transmit and receive branches.

Index Terms—MIMO systems, orthogonal frequency division multiplexing (OFDM), phase noise.

I. INTRODUCTION

RESEARCH concerning multiple-input multiple-output orthogonal frequency division multiplexing (MIMO OFDM) has mainly focussed on systems impaired by additive white Gaussian noise (AWGN). Such analyses will, however, not suffice when implementing a wireless system based on the MIMO OFDM concept. Studies, e.g., [1], concerning the implementation of single-input single-output (SISO) OFDM showed that other system related parameters may significantly affect the system performance. Therefore this letter investigates the influence of non-perfect oscillators, i.e., oscillators causing phase noise (PN), on the performance of a multiple antenna OFDM system. Various papers treat the impact of PN on the performance of SISO OFDM systems, e.g., [2]–[4]. A common conclusion is that the effect can be split into a multiplicative part, which is equal for all subcarriers and therefore referred to as common phase error (CPE), and an additive part, which is referred to as inter-carrier interference (ICI).

This letter extends previous analyses to take into account the effect of multiple antennas and radio/analog front-ends at both sides of the communication link. Section II shows the impact of PN on the system model. Section III compares the influence of transmitter (TX) and receiver (RX) PN on the performance of a zero-forcing (ZF) based MIMO receiver. This analysis is carried out for two fading cases, i.e., frequency flat fading and per subcarrier independent Rayleigh fading channels. The derived analytical values for the power of the ICI induced estimation error can be used to derive the probability of error of MIMO OFDM systems experiencing PN. Results for more realistic fading situations, with partial correlation between the channel elements for the subcarriers,

are bounded by the results for these two cases. The analytical results are in Section V compared with experimental results from simulations. The main conclusion is that in the case of independent Rayleigh fading, the impact of the receiver PN depends on the ratio between the number of transmit and receive branches, while this is not the case for frequency flat fading or transmitter PN.

II. SYSTEM MODEL

Consider a MIMO OFDM system with N_t TX and N_r RX antennas, denoted here as a $N_t \times N_r$ system, applying N_c subcarriers. It is assumed that the branches at the TX (and similar for the RX branches) use a common oscillator/frequency synthesizer and, thus, experience the same PN process.

As detailed in [5], the received $N_c N_r \times 1$ frequency domain vector $\hat{\mathbf{x}}$ is given by

$$\hat{\mathbf{x}} = (\mathbf{G}_{\text{RX}} \otimes \mathbf{I}_{N_r}) \hat{\mathbf{H}} (\mathbf{G}_{\text{TX}} \otimes \mathbf{I}_{N_t}) \hat{\mathbf{s}} + \hat{\mathbf{n}}, \quad (1)$$

where \mathbf{I}_N represents the $N \times N$ dimensional identity matrix and \otimes denotes the Kronecker product. The $N_c N_r \times 1$ transmitted MIMO OFDM vector is given by $\hat{\mathbf{s}} = (\hat{\mathbf{s}}_1^T, \hat{\mathbf{s}}_2^T, \dots, \hat{\mathbf{s}}_{N_c}^T)^T$, where $\hat{\mathbf{s}}_k$ denotes the $N_t \times 1$ frequency domain MIMO transmit vector for the k th subcarrier. We consider a situation where the elements of $\hat{\mathbf{s}}$ are independent, which corresponds to a space division multiplexing system. The quasistatic multipath channel, with an average channel amplitude of unity and delay of zero, is modeled in the frequency domain by the $N_c N_r \times N_c N_t$ block diagonal matrix $\hat{\mathbf{H}}$. The k th $N_r \times N_t$ block diagonal element is \mathbf{H}_k , the MIMO channel for the k th subcarrier, whose elements we assume to be i.i.d. The $N_r N_c \times 1$ vector $\hat{\mathbf{n}}$ represents the receiver noise, with i.i.d. zero-mean, complex Gaussian elements with a variance of σ_n^2 .

The matrices \mathbf{G}_{TX} and \mathbf{G}_{RX} model the influence of the TX and RX PN in the baseband system model, respectively. The (k, l) th element of the $N_c \times N_c$ matrix \mathbf{G}_{TX} , and similarly for \mathbf{G}_{RX} , is given by

$$g_q^{\text{TX}} = \frac{1}{N_c} \sum_{i=0}^{N_c-1} \exp(j\theta_{\text{TX}, (N_g+i+1)}) \exp\left(-j\frac{2\pi qi}{N_c}\right), \quad (2)$$

where $q = k - l$, and $\theta_{\text{TX}, n}$ and $\theta_{\text{RX}, n}$ are sampled random variables that represent the transmitter and receiver PN at sample instant n , respectively. Furthermore, N_g is the guard interval (GI) length. Note that without PN, \mathbf{G}_{TX} and \mathbf{G}_{RX} reduce to identity matrices and (1) becomes the well-known system model for AWGN impaired MIMO OFDM systems. Estimates of the transmitted signal can be found by applying MIMO processing to the received signal $\hat{\mathbf{x}}$.

All elements on the diagonal of \mathbf{G}_{TX} and \mathbf{G}_{RX} are equal, i.e., g_0^{TX} and g_0^{RX} , respectively, and have unity amplitude. Since they are on the diagonal, they cause a rotation $g_0 = g_0^{\text{RX}} g_0^{\text{TX}}$ of

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Tim C. W. Schenk, Peter F. M. Smulders, and Erik R. Fledderus are with Eindhoven University of Technology, Eindhoven, The Netherlands (e-mail: {T.C.W.Schenk, P.F.M.Smulders, E.R.Fledderus}@tue.nl).

Xiao-Jiao Tao is with Sony Ericsson Mobile Communications AB, Kista, Sweden (e-mail: Xiao-Jiao.Tao@sonyericsson.com).

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the wanted signals. As this rotation is equal for all carriers, it is often referred to as common phase error (CPE). The other elements in \mathbf{G}_{TX} and \mathbf{G}_{RX} cause interference among carriers, i.e., the ICI $\hat{\xi}$. This property is used to rewrite (1) to $\hat{\mathbf{x}} = g_0 \hat{\mathbf{H}} \hat{\mathbf{s}} + \hat{\xi} + \hat{\mathbf{n}}$, where

$$\hat{\xi} = (\varphi^{\text{RX}} \otimes \mathbf{I}_{N_r}) \hat{\mathbf{H}} (\varphi^{\text{TX}} \otimes \mathbf{I}_{N_t}) \hat{\mathbf{s}} + g_0^{\text{RX}} \hat{\mathbf{H}} (\varphi^{\text{TX}} \otimes \mathbf{I}_{N_t}) \hat{\mathbf{s}} + g_0^{\text{TX}} (\varphi^{\text{RX}} \otimes \mathbf{I}_{N_r}) \hat{\mathbf{H}} \hat{\mathbf{s}}, \quad (3)$$

$\varphi^{\text{TX}} = \mathbf{G}_{\text{TX}} - g_0^{\text{TX}} \mathbf{I}_{N_c}$ and φ^{RX} has a similar structure as φ^{TX} . For the remainder of the analysis, we assume the CPE g_0 can be perfectly removed, e.g., using the method proposed in [5], so the degradation in performance is only caused by the ICI term.

III. INFLUENCE OF ICI ON THE ESTIMATED SYMBOLS

When the number of carriers is chosen to be large in comparison to the size of the channel dispersion, the MIMO processing can be applied per subcarrier, since the channel experienced at a subcarrier can be regarded frequency flat. Here zero-forcing (ZF) processing is considered, which multiplies the received signal vector with the pseudo-inverse of the estimated channel matrix. For convenience the channel matrix is assumed to be perfectly known at the RX.

In case of PN at both TX and RX, the estimated version of the transmitted signal $\tilde{\mathbf{s}}$ after ZF processing is given by

$$\tilde{\mathbf{s}} = g_0 \hat{\mathbf{s}} + \hat{\mathbf{H}}^\dagger \hat{\xi} + \hat{\mathbf{H}}^\dagger \hat{\mathbf{n}}, \quad (4)$$

where \dagger denotes the pseudo-inverse of a matrix. The order in which the TX and RX term appear in (3), indicate that TX and RX PN induced ICI differently influence the performance of the estimator. To compare their influence, they are analyzed separately in the remainder of this letter.

A. Transmitter Phase Noise

For the case of only TX PN and where, thus, the RX oscillator is ideal, i.e., $\mathbf{G}_{\text{RX}} = \mathbf{I}_{N_c}$, the term $\hat{\mathbf{H}}^\dagger \hat{\xi}$ represents the ICI induced error in the estimated TX signal and is given by $\hat{\mathbf{H}}^\dagger \hat{\xi} = (\varphi^{\text{TX}} \otimes \mathbf{I}_{N_t}) \hat{\mathbf{s}}$. This error term for the k th carrier Ξ_k is given by $\sum_{i=1, i \neq k}^{N_c} g_{k-i}^{\text{TX}} \mathbf{s}_i$. The power of this error term averaged over N_t antennas for the k th carrier is then given by

$$P_{\Xi_k} = \frac{1}{N_t} \mathcal{E} \{ \Xi_k^H \Xi_k \} = \sigma_s^2 \sum_{i=1, i \neq k}^{N_c} \mathcal{E} \left\{ |g_{k-i}^{\text{TX}}|^2 \right\}, \quad (5)$$

where H denotes the conjugate transpose and the elements of \mathbf{s} are assumed to be i.i.d. distributed with zero-mean and a variance of σ_s^2 .

B. Receiver Phase Noise

For a system only experiencing RX PN, i.e., $\mathbf{G}_{\text{TX}} = \mathbf{I}_{N_c}$, the term $\hat{\mathbf{H}}^\dagger \hat{\xi}$ represents the ICI induced error in the estimated TX signal and is given by $\hat{\mathbf{H}}^\dagger \hat{\xi} = \hat{\mathbf{H}}^\dagger (\varphi^{\text{RX}} \otimes \mathbf{I}_{N_r}) \hat{\mathbf{H}} \hat{\mathbf{s}}$. This error term for the k th carrier Ξ_k is given by $\mathbf{H}_k^\dagger \sum_{i=1, i \neq k}^{N_c} g_{k-i}^{\text{RX}} \mathbf{H}_i \mathbf{s}_i$. The average power of Ξ_k is given by

$$\begin{aligned} P_{\Xi_k} &= \frac{1}{N_t} \mathcal{E} \{ \Xi_k^H \Xi_k \} \\ &= \frac{\sigma_s^2}{N_t} \sum_{i=1, i \neq k}^{N_c} \mathcal{E} \left\{ |g_{k-i}^{\text{RX}}|^2 \right\} \text{tr} \left[\mathcal{E} \left\{ \mathbf{H}_i^H \mathbf{H}_k^\dagger \mathbf{H}_k \mathbf{H}_i \right\} \right]. \end{aligned} \quad (6)$$

The average power of the error term of estimated symbols depends on the interaction between channel elements at different subcarrier locations. Therefore, the channel conditions will influence the power of the error term. To quantify the impact, the analysis is carried out for two special cases: *Case 1*, where the channel is flat Rayleigh faded over the whole system bandwidth; and *Case 2*, where all subcarriers experience independent Rayleigh faded channel elements.

1) *Case 1 - Flat Fading*: For the flat fading case $\mathbf{H}_k = \mathbf{H}_i$, and (6) can, thus, be simplified to

$$P_{\Xi_k} = \sigma_s^2 \sum_{i=1, i \neq k}^{N_c} \mathcal{E} \left\{ |g_{k-i}^{\text{RX}}|^2 \right\}. \quad (7)$$

When comparing (7) to (5), it is clear that for flat fading there is no difference between the impact of TX and RX PN.

2) *Case 2 - Independent Fading*: For the independent fading case the elements of \mathbf{H}_k and \mathbf{H}_i are independent and identical distributed according to the zero-mean, unit variance complex Gaussian distribution. Using these channel properties, (6) can be rewritten to

$$\begin{aligned} P_{\Xi_k} &= \frac{\sigma_s^2}{N_t} \sum_{i=1, i \neq k}^{N_c} \mathcal{E} \left\{ |g_{k-i}^{\text{RX}}|^2 \right\} \text{tr} \left[\mathcal{E} \left\{ [\mathbf{H}_k \mathbf{H}_k^H]^\dagger \right\} \mathcal{E} \left\{ \mathbf{H}_i \mathbf{H}_i^H \right\} \right] \\ &= \sigma_s^2 \text{tr} \left[\mathcal{E} \left\{ [\mathbf{H}_k \mathbf{H}_k^H]^\dagger \right\} \right] \sum_{i=1, i \neq k}^{N_c} \mathcal{E} \left\{ |g_{k-i}^{\text{RX}}|^2 \right\}, \end{aligned} \quad (8)$$

where $[\mathbf{H}_k \mathbf{H}_k^H]^\dagger$ is distributed according to the complex inverse Wishart distribution [6]. For the special case of $N_r > N_t$, the authors of [6] show that $\text{tr} \left[\mathcal{E} \left\{ [\mathbf{H}_k \mathbf{H}_k^H]^\dagger \right\} \right] = N_t / (N_r - N_t)$. When we apply this, (8) is simplified to

$$P_{\Xi_k} = \frac{N_t}{N_r - N_t} \sigma_s^2 \sum_{i=1, i \neq k}^{N_c} \mathcal{E} \left\{ |g_{k-i}^{\text{RX}}|^2 \right\}. \quad (9)$$

When comparing the result of (9) with the one for TX PN in (5), we can conclude that when $N_r > 2N_t$, the RX PN has less impact than TX PN for independent Rayleigh faded channels. When $N_t < N_r < 2N_t$, the RX PN has greater impact than TX PN. For example, for a 2 TX and 3 RX system, $N_t / (N_r - N_t) = 2$, RX PN leads to a twice greater average power level of the error term.

IV. PHASE NOISE MODEL: SAMPLED BROWNIAN MOTION

The average power of Ξ_k , as derived above, depends on the PN process through g_q . Here we define it to be a sampled Brownian motion, a commonly used model for free-running oscillators, which is given by $\theta_{\text{TX},n} = \sum_{i=0}^n \varepsilon_i$, where $\varepsilon_n \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ and $\sigma_\varepsilon^2 = 4\pi\beta T$. Here β denotes the one-sided -3 dB bandwidth of the corresponding Lorentzian spectrum and T is the sample time. Applying the properties of the PN process, $\mathcal{E}\{|g_q|^2\}$ can be calculated. Hereto g_q , as defined in (2), is rewritten for $q \in \{-N_c + 1, \dots, -1, 1, \dots, N_c - 1\}$ using the small angle approximation, which is valid when the -3 dB linewidth β is small as compared to the subcarrier spacing $1/(TN_c)$. When, furthermore, the order of summation is changed, g_q is given by

$$g_q = \frac{j\eta}{N_c} \sum_{n=0}^{N_c-1} \varepsilon_n \sum_{i=0}^n \exp(-j2\pi qn/N_c), \quad (10)$$

where $\eta = \exp\left(j \sum_{l=0}^{N_g+1} \varepsilon_l\right)$. The expected value of the power of g_q can now be rewritten to

$$\mathcal{E}\{|g_q|^2\} = \frac{\sigma_\varepsilon^2 \sum_{n=0}^{N_c-1} \sin^2\left(\frac{qn\pi}{N_c}\right)}{N_c^2 \sin^2\left(\frac{q\pi}{N_c}\right)} = \frac{2\pi\beta T}{N_c \sin^2\left(\frac{q\pi}{N_c}\right)}. \quad (11)$$

Substituting expression (11) into (5), for the case of TX PN, the expression for the average power of the error term due to ICI on the k th carrier can be written as

$$P_{\Xi_k} = \frac{2\sigma_s^2 \pi \beta T}{N_c^2} \sum_{i=1, i \neq k}^{N_c} \sin^{-2}\left(\frac{(k-i)\pi}{N_c}\right). \quad (12)$$

In most systems the number of carriers is chosen to be a power of 2, i.e., $N_c = 2^M$, due to ease of implementation of the discrete Fourier transform (DFT) for this number of carriers. When this is assumed, the sum in (12) can be written as a recursive expression and it is found that it can be rewritten to

$$\begin{aligned} \sum_{i=1, i \neq k}^{2^M} \sin^{-2}\left(\frac{(k-i)\pi}{2^M}\right) &= \sum_{i=1}^{2^M-1} \sin^{-2}\left(\frac{i\pi}{2^M}\right) \\ &= 1 + 2 \sum_{i=1}^{2^M-1} \sin^{-2}\left(\frac{i\pi}{2^M}\right) = \frac{2^{2M}-1}{3}. \end{aligned} \quad (13)$$

Substituting (13) in (12) results in the following expression for the average power of the error term Ξ_k :

$$P_{\Xi_k} = \frac{2\sigma_s^2 \pi \beta T (N_c^2 - 1)}{3N_c}. \quad (14)$$

For a large number of carriers this is well approximated by $\frac{2}{3}\sigma_s^2 \pi \beta T N_c$. Note that this expression no longer depends on the carrier index k .

Similarly, the expressions for the average ICI power in (7) and (9) can be further simplified if the PN is modeled by a sampled Brownian motion. The approximation of P_{Ξ_k} for a large number of carriers for *Case I*, i.e., flat Rayleigh fading, is given by $\frac{2}{3}\sigma_s^2 \pi \beta T N_c$ and for *Case II*, i.e., independent Rayleigh fading, by $\frac{2N_t}{3(N_r - N_t)} \sigma_s^2 \pi \beta T N_c$.

V. NUMERICAL RESULTS

The analytical results of Section IV are in Fig. 1 compared with simulation results from Monte-Carlo simulations. In these simulations the sample time T was 50 ns, the number of subcarriers $N_c = 64$, 64-QAM modulation was applied and $\sigma_s^2 = 1$. The PN is modeled as a sampled Brownian motion. In Fig. 1 the average ICI induced estimation error is plotted as function of the -3 dB bandwidth of the Lorentzian power spectral density of the PN β . The results are given for different combinations of number of TX and RX antennas. Every subcarrier experienced an i.i.d. Rayleigh faded channel in the simulations.

It is clear from Fig. 1 that there is good agreement between the analytical results from Section IV and the simulation results. The discrepancy at high β values can be explained by the fact that the small angle approximation is no longer valid. Here β is no longer small compared to the subcarrier spacing, which is 312.5 kHz in this case.

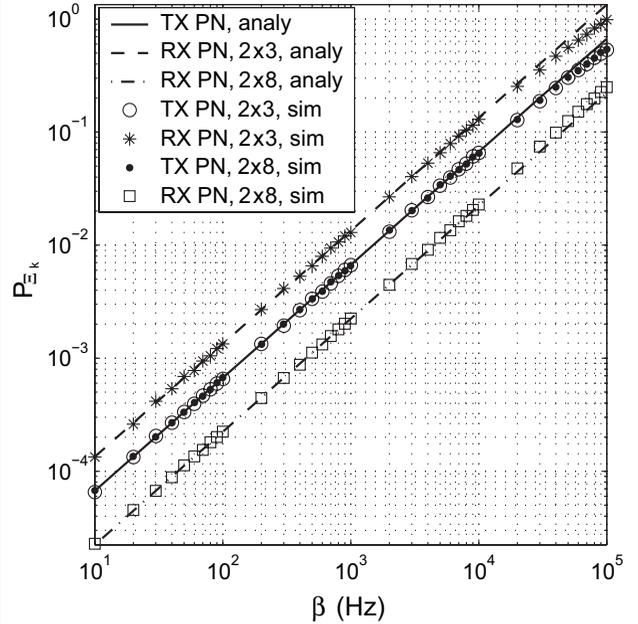


Fig. 1. Average power of error in estimated symbols due to inter-carrier interference as function of the PN corner frequency β , for $T = 50$ ns, $N_c = 64$, $\sigma_s^2 = 1$, 64-QAM modulation. Channel is modeled as independent Rayleigh faded per subcarrier. Analytical (analy) and simulation (sim) results are depicted.

VI. CONCLUSIONS

In this letter the impact of phase noise (PN) on the performance of a wireless system combining multiple-input multiple-output (MIMO) with orthogonal frequency division multiplexing (OFDM) is examined. The influence of transmitter and receiver phase noise is shown to be different. An expression is found for the power of the error term after zero-forcing estimation caused by the ICI term. For TX PN the power of this error term is equal for all MIMO configurations. The power of the error term for RX PN is equal to the TX case in the case of a flat Rayleigh faded channel. For the case of independent Rayleigh faded channels, however, the RX PN is concluded to have less impact than TX PN when $N_r > 2N_t$ and more impact when $N_t < N_r < 2N_t$. Simulation results confirm the results from the analytical study.

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