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A two-step modeling approach for the influence of radio-system impairments in multicarrier MIMO systems

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***Abstract*— In this contribution we introduce a modeling approach for the influence of radio-impairments in (multi-antenna) wireless communication systems. This method, differently from previous approaches, regards two phases: a stage where the impairments are severe and one where the influence is less critical, which enables the inclusion of the impact of power-save-modes and transmitter/receiver mode switching. The error model includes an additive and a rotational part, both implemented in the transmitter as well as in the receiver. As an example, it is shown how phase noise maps onto the introduced error model. As a test case, a multiple-input multiple-output orthogonal frequency division multiplexing (MIMO OFDM) system based on coherent detection is regarded. The system applies zero-forcing as detection method. Analytical expression are derived for the probability of error of such a system impaired by the introduced error model. Finally, numerical results from simulations are shown, which confirm the results from the analytical study.**

I. Introduction

The multicarrier technique orthogonal frequency division multiplexing (OFDM) is adopted in many wireless standards, e.g., the wireless LAN (WLAN) standards IEEE 802.11a and g. The popularity of this technique can be mainly attributed to its high spectral efficiency and ability to deal with frequency selective fading and narrowband interference. Fed by the enormous research effort into multiple antenna systems, architectures are currently proposed which combine OFDM with multiple-input multiple-output (MIMO) techniques. Adding the spatial dimension to OFDM systems largely increases the spectral efficiency, thereby opening the door to high data rate wireless systems.

When implementing systems based on MIMO OFDM, many other impairments than the thoroughly studied additive white Gaussian receiver noise will arise, which can largely affect the performance of the wireless system. Phase noise, I/Q imbalance, limited word length due to fixed point implementation, non-lineair power amplifiers and DC-offset are regarded as the main contributors to bit-error-rate (BER) degradation.

One commonly used measure for the aggregate severity of these imperfections in system design is the *error-vector-magnitude* (EVM) [1], which basically measures the second moment of the error in the estimated symbols. In order to unambiguously relate this measure to the final system performance measure BER, the distribution of the error term resulting from all implementation deficiencies has to be known. Generally, though, it is assumed to be a zero-mean complex Gaussian process. However, when the impairments induced by the radio-frequency (RF) part of the system prevail, this will not be a good approximation, making a performance estimation using EVM far from accurate. Furthermore, the measure does not allow to take into account the non-stationarity of the impairments, which will be present when systems apply power-save modes and switch between transmitter/receiver (TX/RX) modes.

In this paper we, therefore, propose an error-model, that next to an additive part, also has a rotational part. To separate the influence of TX and RX impairments, which will be different when the system experiences a fading channel, the model is implemented at both sides of the wireless channel. To deal with the non-stationary behavior of the RF-impairments, a two-step model is introduced, where the parameters for the error model are different in the first and second part of the reception. For a zero-forcing based MIMO OFDM system, expressions are derived to map this error model to BER.

The outline of the paper is as follows. In Section II the system model of a MIMO OFDM system is introduced. Section III introduces the two-step additive and rotational error-modeling approach. The performance of a MIMO OFDM system experiencing this error model is derived in Section IV. Section V compares these analytical findings to results from simulations. Finally, conclusions are drawn in Section VI.

II. System model

Consider a MIMO OFDM system with N_t TX and N_r RX antennas, denoted here as a $N_t \times N_r$ system, which applies MIMO correction at the receiver. The baseband model for such a system is depicted in Fig. 1.

Let us define the MIMO OFDM vector to be transmitted during a symbol period as $\hat{\mathbf{s}} = \left[\mathbf{s}_0^T, \mathbf{s}_1^T, \ldots, \mathbf{s}_{N_c-1}^T\right]^T$, where $\mathbf{s}_n$ denotes the $N_t \times 1$ frequency domain MIMO transmit vector for the nth subcarrier and N_c represents the number of subcarriers. This vector is transformed to the time domain using the inverse discrete Fourier transform (IDFT); the result is denoted by $\hat{\mathbf{u}}$. A cyclic prefix (CP) is added to the signal, which adds the last $N_t N_g$ elements $\hat{\mathbf{u}}$ on top of $\hat{\mathbf{u}}$. We assume here that the CP is at least equal to the channel impulse response (CIR) length, avoiding inter-symbol-interference (ISI). It is, additionally, assumed that the average total TX power is divided among the TX antennas, such that the covariance matrix of $\hat{\mathbf{s}}$ equals $\sigma_s^2\mathbf{I}$. The TX RF-subsystem impairments are modeled by the block ε_{TX}, which could be additive or multiplicative, or a combination of them. The (impaired) signal

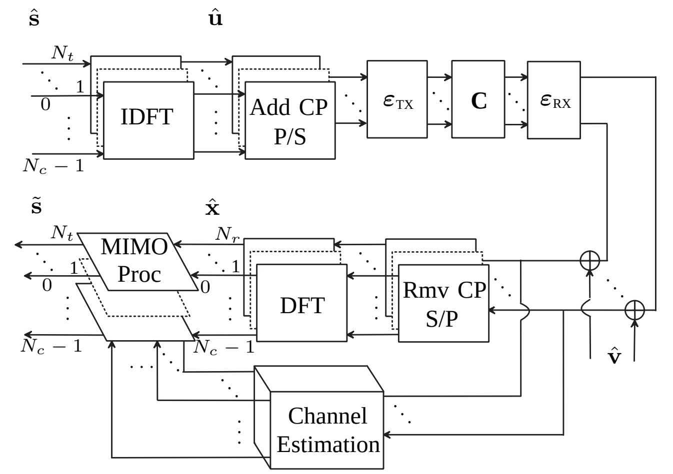

Fig. 1. Baseband model for a MIMO OFDM system with TX and RX impairments applying coherent detection.

is then transmitted through the quasi-static multipath channel $\mathbf{C}$, which has an average attenuation of 1.

At the receiver the signals are impaired by the non-idealities in the RX RF-subsystem, which are modeled in Fig. 1 as ε_{RX}. An extra noise source $\hat{\mathbf{v}}$ is modeled at the RX, which represents the commonly modeled additive white Gaussian receiver noise (AWGN). This enables the separation of the influence of the AWGN and the other impairments. An estimate of the channel matrix $\mathbf{C}$ is acquired, usually enabled by transmission of known symbols preceding the data part. Subsequently, the CP is removed (*Rmv CP*), and the received signal is converted to the frequency domain using the DFT. Without the impairments ε_{TX} and ε_{RX}, this yields

$$\hat{\mathbf{x}} = \hat{\mathbf{H}}\hat{\mathbf{s}} + \hat{\mathbf{n}} \ , \tag{1}$$

where $\hat{\mathbf{H}}$ is the $N_c N_r \times N_c N_t$ channel matrix, which is *block diagonal* since the time domain channel matrix $\mathbf{C}$ is block circulant. The nth $N_r \times N_t$ block diagonal element of $\hat{\mathbf{H}}$ is $\mathbf{H}_n$, the $N_r \times N_t$ MIMO channel of the nth subcarrier. $\hat{\mathbf{n}}$ represents the frequency-domain noise, with i.i.d. zero-mean, complex Gaussian elements and $\hat{\mathbf{v}}$ denotes its time-domain equivalent.

Since the channel is block orthogonal, the MIMO processing can be applied per subcarrier, yielding the $N_c N_t \times 1$ estimate $\tilde{\mathbf{s}}$ of the transmitted symbol vector $\hat{\mathbf{s}}$. Here we regard a zero-forcing (ZF) receiver, which basically multiplies the received signal $\hat{\mathbf{x}}$ with the pseudo-inverse of the estimated channel matrix $\tilde{\mathbf{H}}$, which is given by $\tilde{\mathbf{H}}^{\dagger} = (\tilde{\mathbf{H}}^H \tilde{\mathbf{H}})^{-1} \tilde{\mathbf{H}}^H$. The pseudo-inverse and conjugate transpose are here denoted by † and ^{H}, respectively. Note that for the ZF receiver to work $N_r \geq N_t$.

III. A TWO-STEP ERROR MODELING APPROACH

A. Non-stationarity of the impairments

Many wireless systems, like, e.g., systems based on the IEEE 802.11 standard, operate in packet based transmissions. This is governed by the higher layers, which are based on the TCP/IP protocol. This means that an user terminal (UT), in a down-link scenario, at one point might receive a long burst of packets, but also that at another moment it might not receive packets for long time intervals. Keeping the UT fully functional during this idle time is very power-consuming. Therefore, to design an energy efficient implementation for an UT, it is important to incorporate some kind of *power-save mode*. In this mode the UT periodically looks whether it detects a packet, but is not fully functional to receive the packet. When the UT identifies the beginning of a packet, it switches to the fully functional reception mode. During this power-save mode, many parts in the RF-subsystem are switched off, since they are very power consuming. Since the RF-subsystem includes several filters, it clearly will take a certain time period for the system to settle in a stable mode when it is switched on, making the experienced impairments non-stationary.

Furthermore, in an 802.11 network setup, an acknowledgement for a packet has to be transmitted/received in several μs after reception/transmission of the previous packet. Hereto, the device has to switch between RX/TX mode very rapidly. The allowed time is often referred to as *TX-RX turn around time*. Again, here it would take some time period for the RF-subsystem to settle after the switching.

The above mentioned non-stationarity of the impairments results in a period with severe RF-impairments and a phase where they are less of an issue. This is the reason why we propose to model the impairments in the first part of the reception with other settings than in the second part. However, for simplicity and applicability, we apply the same model in both phases, as proposed in Section III-B, and only change the model parameters.

B. Additive and rotational error model

The proposed error model is schematically represented in Fig. 2. The figure depicts the non-impaired complex symbol s and the impaired versions $\breve{s}$, which lay in a noise cloud around the mean $E(\breve{s})$.

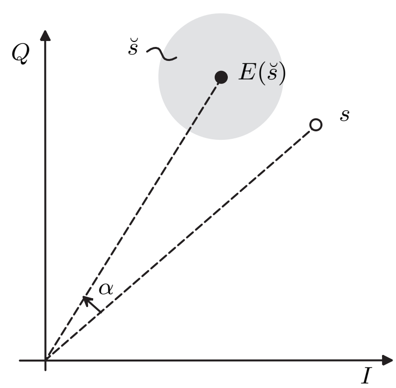

Fig. 2. Schematic representation of the proposed error model.

To allow for more freedom in modeling than inherently assumed by the EVM model, this error model introduces, next to the additive error term, a phase shift of the mean, i.e.,

$E(\breve{s}) = e^{j\alpha} s$. To rewrite this in the notation of Section II, we find that $\breve{\mathbf{s}} = \hat{\boldsymbol{\alpha}}\hat{\mathbf{s}} + \hat{\boldsymbol{\eta}}$, where $\hat{\boldsymbol{\alpha}}$ is a diagonal matrix modeling the phase shift and $\hat{\boldsymbol{\eta}}$ models the additive error matrix with zero-mean entries.

To be able to distinguish between the influence of transmitter and receiver impairments, we model this at both TX and RX side of the communication link. The resulting system model is then given by

$$\hat{\mathbf{x}} = \hat{\boldsymbol{\alpha}}_r \hat{\mathbf{H}} \left(\hat{\boldsymbol{\alpha}}_t \hat{\mathbf{s}} + \hat{\boldsymbol{\eta}}_t\right) + \hat{\boldsymbol{\eta}}_r + \hat{\mathbf{n}} \ , \tag{2}$$

where $\hat{\boldsymbol{\eta}}_t$ and $\hat{\boldsymbol{\eta}}_r$ model the additive TX and RX impairments, respectively. The rotational error is here modeled as a deterministic phase shift, for the TX and RX denoted by the $N_c N_t \times N_c N_t$ diagonal matrix $\hat{\boldsymbol{\alpha}}_t$ and $N_c N_r \times N_c N_r$ diagonal matrix $\hat{\boldsymbol{\alpha}}_r$, respectively. Note that time indices are omitted to increase readability.

For simplicity, we will, in the remainder of this paper, assume that the elements of $\hat{\boldsymbol{\eta}}_t$ and $\hat{\boldsymbol{\eta}}_r$ are zero-mean complex Gaussian distributed and that their covariance matrices are given by $\sigma_t^2 \mathbf{I}$ and $\sigma_r^2 \mathbf{I}$, respectively. Additionally, we assume that the different noise processes are independent. Furthermore, all carriers and MIMO branches are assumed to experience the same phase turn, i.e., $\hat{\boldsymbol{\alpha}}_t$ and $\hat{\boldsymbol{\alpha}}_r$ are given by $e^{j\alpha_t}\mathbf{I}$ and $e^{j\alpha_r}\mathbf{I}$, respectively. Under these assumptions (2) can be rewritten to

$$\hat{\mathbf{x}} = e^{j(\alpha_t + \alpha_r)} \hat{\mathbf{H}} (\hat{\mathbf{s}} + \hat{\boldsymbol{\eta}}_t) + \hat{\boldsymbol{\eta}}_r + \hat{\mathbf{n}} \ . \tag{3}$$

The two-step model can now be incorporated in this model by choosing different parameters for the two periods.

C. Transmission structure

In this section we shortly review the packet structure for a MIMO OFDM system applying coherent detection. Figure 3 conceptually depicts the packet build up for such a system.

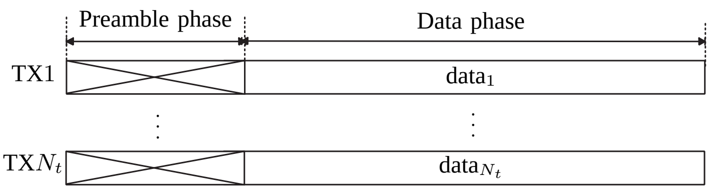

Fig. 3. Packet structure of a MIMO OFDM transmission.

The transmission consists of a *preamble phase* and a *data phase*. The preamble part is used in the receiver to set the automatic gain control (AGC), to do time- and frequency synchronization and to acquire estimates of the MIMO channel matrices for the different subcarriers. To enable simultaneous estimation of the channel elements corresponding to the different TX branches, the preamble has to exhibit some kind of orthogonality. In the data phase the transmitted data is estimated from the received signal, as was described in Section II.

It is clear there is a parallel with the two periods in the RF-impairments, which we identified in Section III-A. Therefore, the preamble phase is now defined to be the part where the RF-impairments are severe and the data phase is the second part, where the influence of the radio-impairments is less.

The influence of the impairments experienced during the preamble now results in errors in the AGC settings, non-ideal synchronization and errors in the estimation of the channel elements. To ease derivation, however, we will assume in the following that the synchronization and AGC are perfect, so that the RF-idealities experienced in the preamble-phase only affect the channel estimates.

D. An example: Phase Noise

As an example, we regard the frequency synthesis (FS) of the RF or intermediate frequency (IF) stages in the radio system. This FS is generally based on a phase lock loop (PLL). It is well known that the locking time of a PLL, when it switches from one frequency to another, is dependent on the frequency jump and the required accuracy. The settling time grows with increasing jump and accuracy.

The phase noise (PN) specifications for MIMO OFDM systems require high accuracy, but the settling time to achieve these in one step would be too high and part of the preamble would not be correctly received. This is why often a two-step approach is chosen. In this method a first stage is applied where the accuracy requirement is low and the locking is fast. Using these settings, the first part of the packet is detected. Then, to increase accuracy and thus PN specifications, the frequency accuracy is increased by changing the loop-filter settings of the PLL in the second stage. Since the frequency jump is much smaller now, the settling time is manageable. As a result the phase noise is worse in the reception of the first part of the packet, mapping perfectly to our two-step model as proposed in Section III-A.

We now regard the influence of PN in a MIMO OFDM system, to see whether the model of Section III-B is applicable. It was shown in [2] that phase noise in multiple-antenna OFDM, similar to what was shown for conventional OFDM in [3], results in a common rotation for all detected carrier on all branches, often referred to as common-phase-error (CPE), and into an additive term due to leakage of the different carriers into each other. The latter effect is often named inter-carrier-interference (ICI). We showed in [2], [4] that for systems experiencing fading channels the structure and influence of the ICI depends on whether it is induced by TX or RX PN.

The CPE can now be modeled in the phase turns of the proposed error model and the ICI in the additive term, where the latter has to be implemented in both TX and RX to model ICI due to TX and RX PN, respectively. Although it is noted that, differently from commonly assumed, the ICI term is not i.i.d. Gaussian distributed, a first approximation of the system performance is found by modeling of the ICI as an i.i.d. zero-mean AWGN impairment, see [5], with a variance as approximated in [4]. This, thus, results in a good mapping to the error model introduced in Section III-B.

IV. Performance evaluation

In a next step, it is important to understand the link between the error model and the final system performance measure BER. Therefore, we derive the probability of error for a QAM/QPSK system experiencing the error model of the previous section.

A. Preamble phase

First we regard the influence in the preamble phase, where the received signal block is given by

$$\hat{\mathbf{X}}_p = e^{j(\alpha_{t,p}+\alpha_{r,p})}\hat{\mathbf{H}}(\hat{\mathbf{S}}_p + \hat{\boldsymbol{E}}_{t,p}) + \hat{\boldsymbol{E}}_{r,p} + \hat{\mathbf{N}} \ , \quad (4)$$

where the subscript p refers to the preamble period and the phase shift is assumed to be constant during the preamble phase. Furthermore, the preamble block is given by $\hat{\mathbf{S}}_p = [\hat{\mathbf{s}}_{p,1}, \dots, \hat{\mathbf{s}}_{p,N_t}]$ and $\hat{\mathbf{X}}_p$, $\hat{\boldsymbol{E}}_{t,p}$, $\hat{\boldsymbol{E}}_{r,p}$ and $\hat{\mathbf{N}}$ have the same structure as $\hat{\mathbf{S}}_p$ and $\hat{\boldsymbol{E}}$ is the block version of $\hat{\boldsymbol{\eta}}$.

The estimate of the channel is now found by multiplication with the pseudo-inverse of the preamble block $\hat{\mathbf{S}}_p$ and is given by

$$\begin{aligned}\tilde{\mathbf{H}} &= \hat{\mathbf{X}}_p\hat{\mathbf{S}}_p^\dagger = e^{j\varphi_p}\hat{\mathbf{H}} + \left(e^{j\varphi_p}\hat{\mathbf{H}}\hat{\boldsymbol{E}}_{t,p} + \hat{\boldsymbol{E}}_{r,p} + \hat{\mathbf{N}}\right)\hat{\mathbf{S}}_p^\dagger \\ &= e^{j\varphi_p}\hat{\mathbf{H}} + \mathbf{E} \ , \end{aligned} \quad (5)$$

where $\varphi_p = \alpha_{t,p} + \alpha_{r,p}$ is the phase error in the estimates of the channel element and $\mathbf{E}$ is the additive error. To have a better estimate of the channel, multiple preamble blocks can be used to average over and improve the channel estimate. Here we assume that M training blocks are used, resulting in a M times smaller variance of $\mathbf{E}$.

B. Data phase

During the data phase the received signal $\hat{\mathbf{x}}$ is given by (3). We estimate the transmitted signal from our received signal by multiplying it with the pseudo-inverse of the estimated channel matrix. The estimates of the transmitted signal are then given by

$$\begin{aligned}\tilde{\mathbf{s}} &= \tilde{\mathbf{H}}^\dagger\hat{\mathbf{x}} \\ &= \left[\hat{\mathbf{H}}(e^{j\varphi_p}\mathbf{I} + \hat{\mathbf{H}}^\dagger\mathbf{E})\right]^\dagger (e^{j\varphi}\hat{\mathbf{H}}(\hat{\mathbf{s}} + \hat{\boldsymbol{\eta}}_t) + \hat{\boldsymbol{\eta}}_r + \hat{\mathbf{n}}) \\ &= e^{-j\varphi_p}(\mathbf{I} + e^{-j\varphi_p}\hat{\mathbf{H}}^\dagger\mathbf{E})^\dagger(e^{j\varphi}(\hat{\mathbf{s}} + \hat{\boldsymbol{\eta}}_t) + \hat{\mathbf{H}}^\dagger(\hat{\boldsymbol{\eta}}_r + \hat{\mathbf{n}})) \\ &\approx e^{-j\varphi_p}(\mathbf{I} - e^{-j\varphi_p}\hat{\mathbf{H}}^\dagger\mathbf{E})(e^{j\varphi}(\hat{\mathbf{s}} + \hat{\boldsymbol{\eta}}_t) + \hat{\mathbf{H}}^\dagger(\hat{\boldsymbol{\eta}}_r + \hat{\mathbf{n}})) \\ &= e^{j(\varphi-\varphi_p)}\hat{\mathbf{s}} + \boldsymbol{\delta} \ , \end{aligned} \quad (6)$$

where we apply an approximation of the pseudo-inverse using the first two terms of the Taylor expansion. Here the additive error in estimation of $\hat{\mathbf{s}}$ is given by

$$\begin{aligned}\boldsymbol{\delta} &= e^{j(\varphi-\varphi_p)}\hat{\boldsymbol{\eta}}_t + e^{-j\varphi_p}\hat{\mathbf{H}}^\dagger(\hat{\boldsymbol{\eta}}_r + \hat{\mathbf{n}}) - e^{j(\varphi-2\varphi_p)}\hat{\mathbf{H}}^\dagger\mathbf{E}\hat{\mathbf{s}} \\ &\quad - e^{j(\varphi-2\varphi_p)}\hat{\mathbf{H}}^\dagger\mathbf{E}\hat{\boldsymbol{\eta}}_t - e^{-j2\varphi_p}\hat{\mathbf{H}}^\dagger\mathbf{E}\hat{\mathbf{H}}^\dagger(\hat{\boldsymbol{\eta}}_r + \hat{\mathbf{n}}) \ . \end{aligned} \quad (7)$$

It is now easily found that $\boldsymbol{\delta}$ is a zero-mean Gaussian variable and that for small errors in the estimates of the channel elements, its covariance matrix $\boldsymbol{\Omega}$ is well approximated by

$$\begin{aligned}\boldsymbol{\Omega} &\approx \left(\sigma_t^2 + \tfrac{\sigma_{t,p}^2}{M}\right)\mathbf{I} + \left(\tfrac{M+1}{M}\sigma_n^2 + \sigma_r^2 + \tfrac{\sigma_{r,p}^2}{M}\right)(\hat{\mathbf{H}}^H\hat{\mathbf{H}})^{-1} \\ &= \sigma_k^2\mathbf{I} + \sigma_l^2(\hat{\mathbf{H}}^H\hat{\mathbf{H}})^{-1} \ . \end{aligned} \quad (8)$$

Regarding (6), it can be concluded that the estimated symbols are given by a rotated version of the transmitted symbols plus an additive error term.

C. Probability of error

In calculation of the probability of error, we first regard the additive error terms. These result in a signal-to-noise ratio (SNR) for the n_tth branch and the nth subcarrier, affected by a given channel, imperfect channel knowledge and TX and RX impairments, which is given by

$$\begin{aligned}\rho &= \frac{\sigma_s^2}{\sigma_k^2 + \sigma_l^2[(\mathbf{H}_n^H\mathbf{H}_n)^{-1}]_{n_tn_t}} \\ &= \left[\frac{1}{\rho_k} + \frac{N_t[(\mathbf{H}_n^H\mathbf{H}_n)^{-1}]_{n_tn_t}}{\rho_l}\right]^{-1} = \frac{\rho_k\rho_\lambda}{\rho_k + \rho_\lambda} \ , \end{aligned} \quad (9)$$

where $[\mathbf{A}]_{mm}$ denotes the mth diagonal element of matrix $\mathbf{A}$ and where the transmitter and receiver SNR are given by

$$\rho_k = \frac{\sigma_s^2}{\sigma_k^2} = \frac{\sigma_s^2}{\sigma_t^2 + \frac{\sigma_{t,p}^2}{M}} \ , \quad (10)$$

and

$$\rho_l = \frac{N_t\sigma_s^2}{\sigma_l^2} = \frac{N_t\sigma_s^2}{\frac{M+1}{M}\sigma_n^2 + \sigma_r^2 + \frac{\sigma_{r,p}^2}{M}} \ , \quad (11)$$

respectively. When the channel matrix $\mathbf{H}$ has i.i.d. complex Gaussian elements, often referred to as a Rayleigh faded channel, ρ_λ in (9) is chi-squared distributed with $2(N_r - N_t + 1)$ degrees of freedom.

The symbol error rate (SER), for only the additive error terms, for the n_tth branch and nth subcarrier of an uncoded system is then found by [6]

$$P_e = \int_0^\infty P_{e,M}(\rho)p(\rho)d\rho \ , \quad (12)$$

where the $P_{e,M}(\rho)$ denotes the approximation of the SER for a M-QAM constellation and is given by $aQ(\sqrt{b\rho})$. In this expression $a = 4(1 - 1/\sqrt{M})$ and $b = 3/(M-1)$. The distribution of ρ is denoted by $p(\rho)$. The average SER is now found by averaging P_e over the different subcarriers and branches. The average BER is then found by dividing the SER by $\log_2(M)$.

When we also take into account the influence of the phase turn in (6), i.e., $e^{j(\varphi-\varphi_p)} = e^{j\phi}$, we have to replace $P_{e,M}$ in (12) by $P_{e,M}^\phi$. Here $P_{e,M}^\phi$ denotes the probability of error for a receiver where the reference at the decision device is phase shifted by ϕ. For QPSK modulation the SER with a phase shift of ϕ is given by [7]

$$P_{e,\mathrm{QPSK}}^\phi = Q\left(\sqrt{\rho}c_\phi\right) + Q\left(\sqrt{\rho}d_\phi\right) - Q\left(\sqrt{\rho}c_\phi\right)Q\left(\sqrt{\rho}d_\phi\right) \ , \quad (13)$$

where $c_\phi = \sqrt{2}\sin\left(\frac{\pi}{4} - \phi\right)$ and $d_\phi = \sqrt{2}\cos\left(\frac{\pi}{4} - \phi\right)$.

V. Numerical results

In this section results from the SER expressions derived in the previous section are compared with results from Monte Carlo simulations. As a test case, a MIMO extension of the IEEE 802.11a WLAN standard was studied. The applied parameters are: modulation is QPSK, bandwidth is 20 MHz, number of subcarriers N_c = 64 and no coding is applied.

We now assume that the impairments during the preamble period are q times worse than in the data phase, i.e., $\sigma_{t,p}^2 = q\sigma_t^2$ and $\sigma_{r,p}^2 = q\sigma_r^2$. We depict the BER as function of the SNR for a system with only additive AWGN receiver noise, i.e., $\rho_0 = N_t\sigma_s^2/\sigma_n^2$, to clearly show the influence of the impairments. There is only one block used for estimation of the channel matrices, thus M=1.

Figure 4 shows the results for a 2×2 system (in dashed lines) and 2×4 system (in solid lines) experiencing additive white Gaussian receiver noise and a phase shift ϕ. The additive receiver and transmitter impairments are put to zero, i.e., $\sigma_r = \sigma_t = 0$. It is obvious from Fig. 4 that there is a close agreement between the theoretical and simulation results.

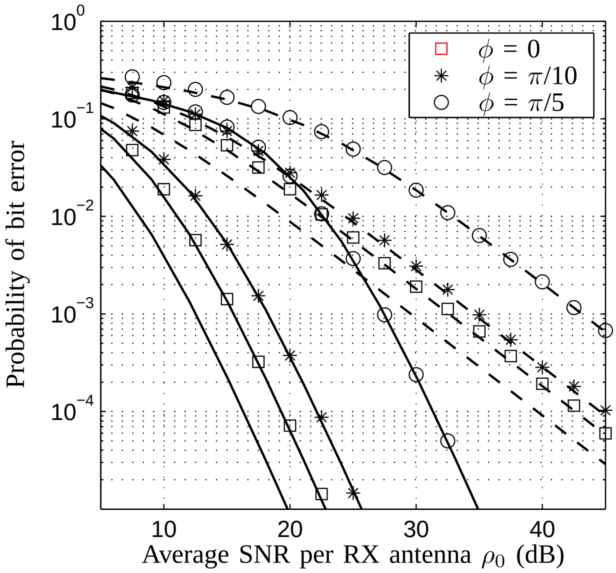

Fig. 4. BER performance of a 2×2 (−−) and 2×4 (—) system for different phase shifts ϕ. Theoretical results are depicted by lines and the simulations results by markers. The curves for a system with perfect channel estimation are given as reference (without markers).

It can be concluded from Fig. 4 that phase shifts result in bigger degradation for the 2×4 than the 2×2 system. Furthermore, degradation is small for angles smaller than π/10, but increases rapidly for bigger angles.

In Fig. 5 results are depicted for the same MIMO configurations, for systems experiencing additive TX and RX impairments. The impairments during the preamble phase are twice as high as during the data phase ($q = 2$) and there is no phase error ($\phi = 0$).

It can be concluded from this figure that for the regarded impairments, flooring is visible for both MIMO configurations. It is clear from the figure, that the flooring is independent of the MIMO configuration for additive TX impairments, but depends on this configuration for the RX impairments, for this fading environment. Discrepanties between simulation and analytical results can be explained by the way the BER is calculated from the SER, assuming only one bit error per symbol, which is not accurate at low SNR, and by approximations made in (6) and (8), which are not accurate for high values of σ_t and σ_r.

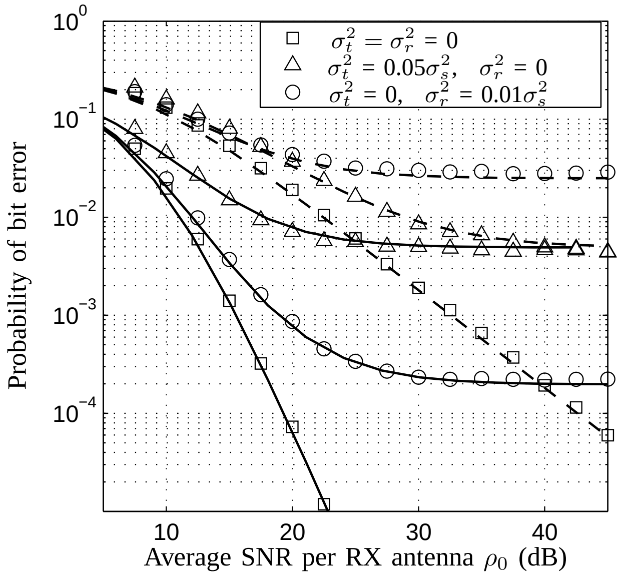

Fig. 5. BER performance of a 2×2 (−−) and 2×4 (—) system for different additive TX/RX impairments. Parameters are $q = 2$ and $\phi = 0$. Theoretical results are depicted by lines and the simulations results by markers.

VI. Conclusions and discussion

In this paper we proposed an approach for modeling the influence of radio-impairment in wireless systems. The model incorporates an *additive* and a *rotational error* term, which are implemented for both transmitter and receiver. The rotational part is found to be important, since it results in large errors, e.g., shown in Fig. 4. Furthermore, this method models the influence of the impairments in two-steps: a first period with severe imperfections and a second period with less influence of the non-idealities. It is shown, both analytically and numerically, how the error model maps to bit-error-rate for a ZF-based multiple-antenna OFDM system. The mapping of impairments, other than phase noise, onto the error model remains a topic for further study.

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