The Application of Spatial Shifting for Peak-to-Average Power Ratio Reduction in MIMO OFDM Systems

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***Abstract*—This paper investigates the application of spatial shifting (SS) of partial transmit sequences (PTSs) in multiple-input multiple-output OFDM. The technique rearranges the transmit (TX) vector in such a way that subparts are transmitted on those TX branches that result in the lowest overall peak-to-average power ratio. The application of different subcarrier grouping schemes and the combination of SS with phase shifting (PS) of the PTSs is investigated. Furthermore, a transparant extension of the techniques, without extra signalling overhead, is proposed. Numerical results prove the effectiveness of the SS and the combined SS/PS approach.**

I. Introduction

The advantages of multiple-input multiple-output orthogonal frequency division multiplexing (MIMO OFDM) as basis of next generation high data-rate broadband systems have been shown in numerous publications over the last few years. Important challenges, however, remain in the efficient implementation of these kind of systems. In this paper we regard a solution to one of the inherent disadvantages of using OFDM: the time-domain signals exhibit a high peak-to-average power ratio (PAPR), which can result in clipping and non-lineair distortion in the power amplifier (PA). A conventional solution to this problem is to apply a large back-off (BO) when feeding the signals to the PA. This is, however, a very inefficient solution, since the power efficiency of the PA decreases rapidly with increasing BO.

Therefore, several more efficient baseband methods were previously presented to reduce the PAPR of single-input single-output (SISO) OFDM transmissions. Some methods try to reduce the PAPR by deliberately clipping the signal in the TX baseband, which however can result in spectral regrowth. When instead of clipping, peak windowing is applied, see e.g. [1], [2], this disadvantage can largely be overcome, however, at the cost of a higher distortion within the signal bandwidth. Other techniques apply coding on top of the OFDM-processing to reduces the PAPR, see e.g. [3], [4]. These codes, however, often do not achieve the same error correction performance as codes specifically designed for this purpose and can thus result in decreased data rate. Yet another approach is to apply different phase-shifts to the data on the different carriers such that the time-domain signal has a minimum PAPR, see e.g. [5]–[8], or to apply different signal representations for the same data and choosing the one with the lowest PAPR, see e.g. [9]. Although these techniques achieve a high PAPR reduction, extra computational complexity is required and (a small amount of) overhead is introduced, since the chosen phase shifts/signal representation, often referred to as side information (SI), has to be transmitted to the receiver (RX).

The application of the techniques of [5] and [9] to MIMO OFDM was studied in [10] and [11]. The authors compare the results for the case of applying the techniques per transmitter (TX) branch separately with a simplified approach, where the optimization is carried out jointly over all TX branches. Although the latter requires the transmission of less SI, the achieved PAPR reduction is also smaller.

Recently the authors proposed a technique called *spatial shifting* (SS) to reduce the PAPR in MIMO OFDM systems [12]. In this technique the extra degree of freedom provided by MIMO is exploited to reduce the PAPR. It is based on the rearrangement of the TX vector in such a way that subparts are transmitted on those TX branches, resulting in the lowest overall PAPR. In this paper several extensions to the concept of SS are proposed and evaluated.

The layout of this paper is as follows. First Section III discusses the SS technique. To further increase the PAPR reduction performance of SS, the combination of SS with the phase shifting (PS) procedure of [5] is proposed in Section IV. Section V regards the application of these PAPR reduction approaches for different subcarrier grouping schemes. To reduce the overhead, the application of SS in a transparent mode, i.e., without the transmission of SI, is illustrated in Section VI. The performance of the different algorithms is evaluated in Section VII by means of Monte Carlo simulations. Finally, Section VIII presents the conclusions of the work.

II. Multiple antenna OFDM system

Consider a MIMO OFDM system applying N_t TX branches and N subcarriers. For such a system the $NN_\mathrm{t} \times 1$ transmit vector corresponding to the mth time-domain symbol is given by

$$\mathbf{s}_m = \left(\mathbf{I}_{N_\mathrm{t}} \otimes \mathbf{F}_N^{-1}\right) \mathbf{S}_m \ , \qquad (1)$$

where the N_t-dimensional identity matrix is denoted by $\mathbf{I}_{N_\mathrm{t}}$, $\mathbf{F}_N$ represents the N-dimensional discrete Fourier transform (DFT) matrix, the (k,l)th element of which is given by $\exp(-j2\pi\frac{kl}{N})$, and $\otimes$ denotes the direct matrix or Kronecker product. The coded, interleaved and (complex) modulated data symbols are contained in the $NN_\mathrm{t}\times 1$ frequency domain vector $\mathbf{S}_m$. The vector $\mathbf{s}_m$, and similarly $\mathbf{S}_m$, is given by a stacking of the OFDM vectors for the different TX branches, which can be expressed as

$$\mathbf{s}_m = \left[\mathbf{s}_{m,1}^T, \mathbf{s}_{m,2}^T, \ldots, \mathbf{s}_{m,N_\mathrm{t}}^T\right]^T \ , \qquad (2)$$

where $\mathbf{s}_{m,n_\mathrm{t}}$ denotes the $N \times 1$ subvector for the n_tth branch.

Subsequently, the PAPR of the mth symbol on the n_tth TX branch can be defined as

$$\mathrm{PAPR}_{m,n_\mathrm{t}} = \frac{\max\left(\mathbf{s}_{m,n_\mathrm{t}} \circ \mathbf{s}^*_{m,n_\mathrm{t}}\right)}{\mathbb{E}\left(\mathbf{s}_{m,n_\mathrm{t}} \circ \mathbf{s}^*_{m,n_\mathrm{t}}\right)} \ , \quad (3)$$

where * and $\circ$ denote complex conjugation and element-wise multiplication, respectively. The functions $\mathbb{E}(\mathbf{y})$ and $\max(\mathbf{y})$ produce the expected value and maximum element of the input vector $\mathbf{y}$ as their output, respectively.

III. Spatial Shifting

Recently the authors proposed a PAPR reduction approach named spatial shifting [12], the basics of which are repeated here for convenience. In SS the datacarriers of the mth OFDM symbol are subdivided into P disjoint groups, referred to as partial transmit sequences (PTSs), similar as was proposed in [5] for conventional OFDM.

The pth subcarrier group for the n_tth TX branch can then be defined as the $N \times 1$ vector

$$\mathbf{S}^{(p)}_{m,n_\mathrm{t}} = \mathbf{\Psi}_p \mathbf{S}_{m,n_\mathrm{t}} \ , \quad (4)$$

where $\mathbf{\Psi}_p$ is a diagonal matrix, where the diagonal elements are 1 for carriers belonging to subcarrier group p and all others are zero. The construction of $\mathbf{\Psi}_p$ is treated in Section V. The resulting time domain pth PTS is then given by

$$\mathbf{s}^{(p)}_{m,n_\mathrm{t}} = \mathbf{F}^{-1}_N \mathbf{S}^{(p)}_{m,n_\mathrm{t}} \ . \quad (5)$$

The vector containing the pth PTS for all TX streams is denoted by

$$\mathbf{s}^{(p)}_m = \left[\mathbf{s}^{(p)}_{m,1}{}^T, \mathbf{s}^{(p)}_{m,2}{}^T, \ldots, \mathbf{s}^{(p)}_{m,N_\mathrm{t}}{}^T\right]^T \ . \quad (6)$$

The original time domain transmit vector is then simply found by a summation over the P PTSs, resulting in

$$\mathbf{s}_{m,n_\mathrm{t}} = \sum_{p=1}^{P} \mathbf{s}^{(p)}_{m,n_\mathrm{t}} = \mathbf{F}^{-1}_N \sum_{p=1}^{P} \mathbf{\Psi}_p \mathbf{S}_{m,n_\mathrm{t}} \ , \quad (7)$$

where by definition $\sum_{p=1}^{P} \mathbf{\Psi}_p = \mathbf{I}_N$.

Alternative transmit vectors representing the same data can now be constructed by mutually interchanging the corresponding PTSs between the different TX branches, provided that the same subcarrier grouping is applied on all branches. This exchanging of the PTSs between the spatial streams is here referred to as *spatial shifting* (SS). The mth transmit vector for the pth PTS (6) after SS can be written as

$$\mathbf{s}^{(p)}_{m,\mathbf{c}_p} = \left[\mathbf{s}^{(p)}_{m,c_{p,1}}{}^T, \mathbf{s}^{(p)}_{m,c_{p,2}}{}^T, \ldots, \mathbf{s}^{(p)}_{m,c_{p,N_\mathrm{t}}}{}^T\right]^T \ , \quad (8)$$

where the SS vector $\mathbf{c}_p = [c_{p,1}, \ldots, c_{p,N_\mathrm{t}}]$ is found by reshuffling the vector $[1, \ldots, N_\mathrm{t}]$.

The total $NN_\mathrm{t} \times 1$ MIMO OFDM vector after SS is then given by

$$\tilde{\mathbf{s}}_{m,\mathbf{c}} = \sum_{p=1}^{P} \mathbf{s}^{(p)}_{m,\mathbf{c}_p} \ , \quad (9)$$

where the SS vector for all PTSs is given by $\mathbf{c} = [\mathbf{c}_1, \ldots, \mathbf{c}_P]$. In the case of P PTSs and N_t TX branches, there are N_t^{P-1} unique realizations of $\tilde{\mathbf{s}}_{m,\mathbf{c}}$.

To achieve a low PAPR on all TX branches, we now want to select the transmit vector $\tilde{\mathbf{s}}_{m,\mathbf{c}}$ that exhibits the lowest peak power averaged over the TX branches. When we apply this as criterion to determine the SS vector for the mth MIMO OFDM symbol $\tilde{\mathbf{c}}_m$, we find that is found by

$$\tilde{\mathbf{c}}_m = \arg\min_{\mathbf{c}} \left(\sum_{n_\mathrm{t}=1}^{N_\mathrm{t}} \max\left(\tilde{\mathbf{s}}_{m,\mathbf{c},n_\mathrm{t}} \circ \tilde{\mathbf{s}}^*_{m,\mathbf{c},n_\mathrm{t}}\right)\right) \ , \quad (10)$$

where $\tilde{\mathbf{s}}_{m,\mathbf{c},n_\mathrm{t}}$ is the n_tth $N \times 1$ subvector of $\tilde{\mathbf{s}}_{m,\mathbf{c}}$ and where $\arg\min(.)$ produces the argument for which the expression is minimized.

IV. Combining SS and PS

When the above treated SS technique is combined with phase shifting (PS), as originally proposed for SISO systems in [5], a further improvement in PAPR performance can be achieved. In this combined technique the mutually (spatially) interchanged PTSs are, before being summed, multiplied by a set of phase shifts, for the pth PTS denoted by $\mathbf{b}_p = [b_{1,p}, \ldots, b_{N_\mathrm{t},p}]$.

The mth transmit vector for the pth PTS after SS and PS can be written as

$$\mathbf{s}^{(p)}_{m,\mathbf{b}_p,\mathbf{c}_p} = \left[b_{1,p}\ \mathbf{s}^{(p)}_{m,c_{p,1}}{}^T, \ldots, b_{N_\mathrm{t},p}\ \mathbf{s}^{(p)}_{m,c_{p,N_\mathrm{t}}}{}^T\right]^T \ . \quad (11)$$

The resulting MIMO OFDM vector is then found by summing the vectors for the different PTSs in (11) and given by

$$\tilde{\mathbf{s}}_{m,\mathbf{b},\mathbf{c}} = \sum_{p=1}^{P} \mathbf{s}^{(p)}_{m,\mathbf{b}_p,\mathbf{c}_p} \ , \quad (12)$$

where the PS vector for all PTSs is given by $\mathbf{b} = [\mathbf{b}_1, \ldots, \mathbf{b}_P]$.

The number of possible phases is chosen as a compromise between complexity and achievable PAPR reduction. Low complex implementation can be found for two or four phases, for which $b_{n_\mathrm{t},p} \in \{-1, 1\}$ and $b_{n_\mathrm{t},p} \in \{\pm 1, \pm j\}$, respectively, since for these cases no multiplications have to be performed, merely changing of the sign and interchanging of the real and imaginary part.

The PS and SS vectors for the mth symbol are selected as the combination that minimizes the peak power averaged over the TX branches and is thus given by

$$\{\tilde{\mathbf{b}}_m, \tilde{\mathbf{c}}_m\} = \arg\min_{\{\mathbf{b},\mathbf{c}\}} \left(\sum_{n_\mathrm{t}=1}^{N_\mathrm{t}} \max\left(\tilde{\mathbf{s}}_{m,\mathbf{b},\mathbf{c},n_\mathrm{t}} \circ \tilde{\mathbf{s}}^*_{m,\mathbf{b},\mathbf{c},n_\mathrm{t}}\right)\right) . \quad (13)$$

The combined SS/PS technique, as described above, is summarized in the block diagram of Fig. 1.

A less computational complex suboptimal solution is found by applying the same PS for all TX antennas, as was also proposed in [10]. The resulting PS vector $\mathbf{b}$ then only contains

P elements and the resulting MIMO OFDM vector is given by

$$\tilde{\mathbf{s}}_{m,\mathbf{b},\mathbf{c}} = \sum_{p=1}^{P} b_p \, \mathbf{s}^{(p)}_{m,\mathbf{c}_p} \quad . \tag{14}$$

The optimal $\mathbf{b}$ and $\mathbf{c}$ are again found by the use of (13).

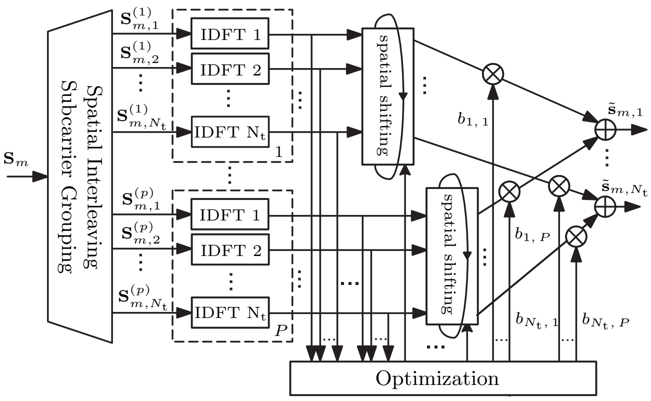

Fig. 1. Block diagram of a MIMO OFDM TX combining SS and PS.

V. Subcarrier grouping schemes

The authors of [6] studied the influence of different grouping schemes on the performance of PS for single-antenna OFDM. The regarded schemes where: 1) grouping of neighboring carriers (*NC*), 2) subcarrier interleaved (*SInt*) grouping and 3) pseudo-random (*PR*) grouping.

For the first two schemes the subcarrier grouping matrix $\mathbf{\Psi}_p$ in (4) can be written as

$$\text{NC:} \quad \mathbf{\Psi}_p = \text{diag}(\mathbf{0}_{(p-1)\frac{N}{P}}, \mathbf{1}_{\frac{N}{P}}, \mathbf{0}_{(P-p)\frac{N}{P}}) \ , \tag{15}$$

$$\text{SInt:} \quad \mathbf{\Psi}_p = \text{diag}(\mathbf{1}_{\frac{N}{P}} \otimes [\mathbf{0}_{p-1}, 1, \mathbf{0}_{P-p}]) \ , \tag{16}$$

for $p = 1, \ldots, P$, respectively. Here $\mathbf{0}_L$ and $\mathbf{1}_L$ are the $1 \times L$ all-zeroes and all-ones vector, respectively. Note that we here assumed N to be a multiple of P.

For the PR grouping the carrier grouping matrix $\mathbf{\Psi}_p$ is constructed in a random method, however, with the following two constraints: 1) N/P of the diagonal elements have to equal 1 and the others have to equal 0, 2) the set of P matrices must be constructed such that $\sum_{p=1}^{P} \mathbf{\Psi}_p = \mathbf{I}_N$.

The effect of the different subcarrier grouping schemes on the PAPR reduction performance is studied in Section VII.

VI. Side information / Transparency

It can be concluded from (10) and (13), that the above proposed techniques have to be applied per MIMO OFDM symbol to perform optimally. For the RX to correct for the SS and PS applied at the TX, SI has to be transmitted containing which PS and SS vectors $\tilde{\mathbf{b}}_m$ and $\tilde{\mathbf{c}}_m$ were chosen. For a high number of PTSs, TX antennas or possible phase shift the overhead clearly might become high. Hereto the number of possible phase shift and PTSs can be minimized or, alternatively, the optimization of (10) and (13) can be carried out jointly for a number of K symbols. The chosen PS and SS vectors are then given by

$$\{\tilde{\mathbf{b}}, \tilde{\mathbf{c}}\} = \arg \min_{\{\mathbf{b},\mathbf{c}\}} \left(\sum_{m=1}^{K} \sum_{n_\text{t}=1}^{N_\text{t}} \max \left(\tilde{\mathbf{s}}_{m,\mathbf{b},\mathbf{c},n_\text{t}} \circ \tilde{\mathbf{s}}^*_{m,\mathbf{b},\mathbf{c},n_\text{t}} \right) \right) . \tag{17}$$

Although this will clearly lead to less reduction in PAPR, it will result in a solution which is less computational complex and has an overhead which is K times lower.

For some systems, however, it might be beneficial to design a fully transparent solution, where the RX needs no information about the applied SS and PS vector. Such a solution is found when the same SS/PS vectors $\{\mathbf{b}, \mathbf{c}\}$ are applied to all symbols within a MIMO OFDM packet, thus also to the part of the transmission used for MIMO channel estimation. The influence of the SS/PS applied at the TX is now included in the *effective* MIMO channel estimate found at the RX. When the RX compensates for this effective MIMO channel, the SS/PS is also removed.

From the viewpoint of the RX the SS method can be seen as effectively interchanging the columns of the physical MIMO channel matrix and the PS method can be seen as phase shifting of the physical MIMO channel matrix. In that way the method is transparent, meaning there is no need to transmit SI, which enables the application of this technique without standardization. The resulting PAPR reduction will, however, be less effective than in the SI case since the peak minimization is done jointly for the full packet. The application seems thus primarily interesting for transmissions with short packets.

VII. Numerical results

The performance of the proposed PAPR reduction methods for MIMO OFDM systems was tested by the use of Monte Carlo simulations, the results of which are reported in this section. The parameters of the simulations are N = 64 carriers, number of TX branches N_t = {2,4} and QPSK modulation is used. The performance is given in terms of the complementary cumulative distribution function (CCDF) of the PAPR, which indicates the probability that the PAPR is larger then PAPR_0. When clipping occurs at power level PAPR_0, for a normalized signal power, the CCDF values indicate the probability of clipping. In all figures the reference curve, i.e., the PAPR without SS or PS, is indicated by $P = 1$.

Figure 2 reports PAPR results for the SS method with SI, as resulting from the optimization in (10). The results are given for 2 and 4 TX branches and for 2 and 4 PTSs. The pseudo-random (PR) carrier grouping scheme is applied. It can be concluded from this figure that at a clipping probability of 10^{-3} and for 2 TX branches, 0.9 dB and 2.2 dB PAPR reduction is achieved for $P = 2$ and $P = 4$, respectively. For 4 TX branches the reduction is 1.4 dB and 2.6 dB, respectively.

The results for the combined SS/PS method are depicted in Fig. 3, where the same phase shifts are applied for the corresponding PTSs on the different TX branches, as in (14),

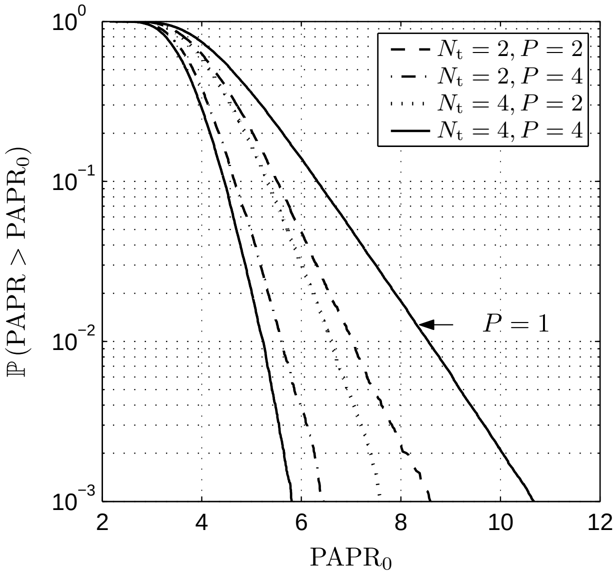

Fig. 2. CCDF of the PAPR for the SS method for the PR subcarrier grouping scheme.

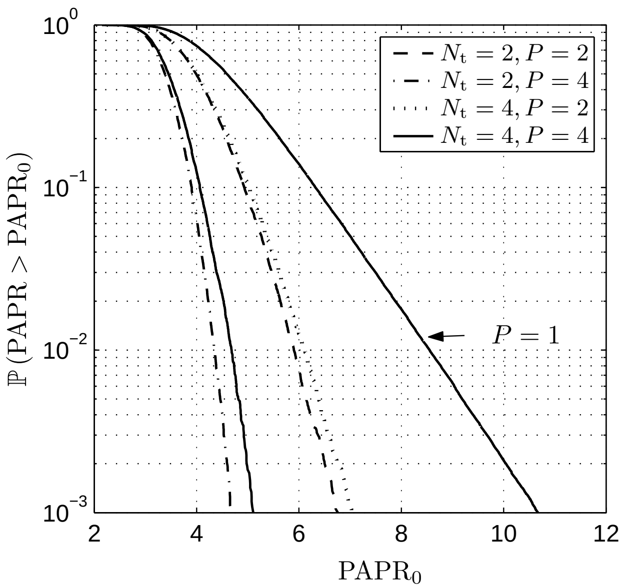

Fig. 3. CCDF of the PAPR for the combined SS/PS method for the PR subcarrier grouping scheme.

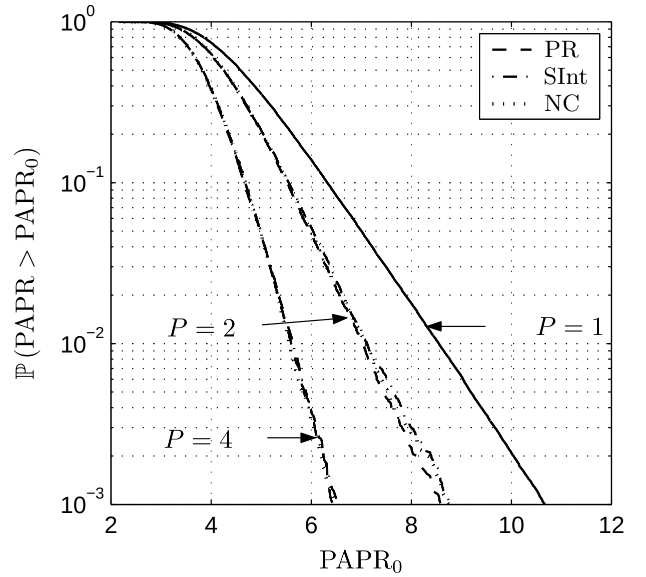

Fig. 4. CCDF of the PAPR for the SS method for the different subcarrier grouping schemes, for $N_t = 2$.

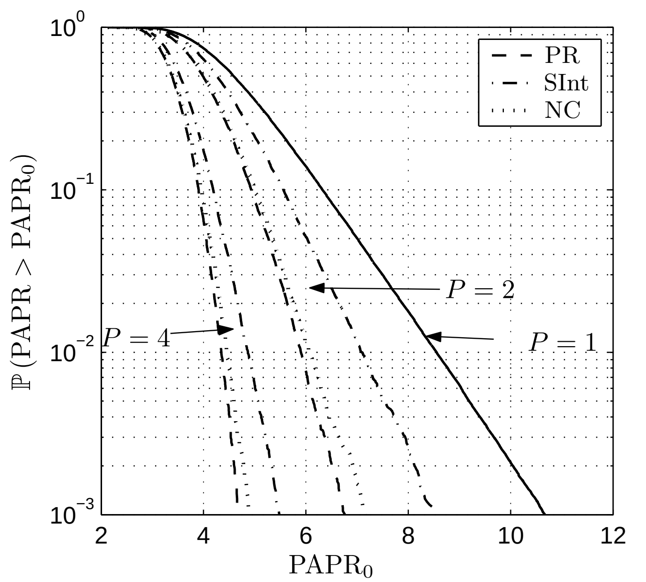

Fig. 5. CCDF of the PAPR for the combined SS/PS method for the different subcarrier grouping schemes, for $N_t = 2$.

and $b_p \in [-1, 1]$. In this way the additional complexity due to the PS method is low. It can be concluded from the results that at a clipping probability of 10^{-3} and for 2 TX branches, 1.9 dB and 3.6 dB PAPR reduction is achieved for $P = 2$ and $P = 4$, respectively. For 4 TX branches and the reduction is 1.7 dB and 3.2 dB, respectively. Clearly improved performance is achieved compared to solely applying SS. It is interesting to note that a better performance is achieved for $N_t = 2$ than for $N_t = 4$, which can be attributed to fact that common phase shifts are applied to the PTSs on all TX branches.

The influence of the different subcarrier grouping schemes as presented in Section V, is evaluated in Fig. 4 and 5. Figure 4 depicts the CCDF of the PAPR resulting from the SS method for a system with 2 TX branches, applying 2 and 4 PTSs. Although from [6] the results were anticipated to differ for the various grouping schemes, it can be concluded that the performance of SS does not depend on the selected grouping.

For the combined SS/PS scheme, the results of which are in Fig. 5, the PAPR performance does depend on the selected subcarrier grouping. Clearly the subcarrier interleaved (SInt) scheme performs worst and the pseudo-random (PR) grouping yields the best performance. This confirms the findings of the authors of [6] for SISO OFDM.

Finally, Fig. 6 presents the PAPR results for the transparent SS and SS/PS method with $P = 2$ and $P = 4$ for a system with 2 TX branches. It is noted that the PAPR for the transparent method is calculated for the entire 10 OFDM symbol packet. It can be concluded from the results that at a clipping probability of 10^{-3} and for the SS method, 0.9 dB and 1.8 dB PAPR reduction is achieved for $P = 2$ and $P = 4$, respectively. For the SS/PS method the reduction is 1.4 dB and 2.7 dB, respectively. This enforces the conclusion that, although its PAPR performance is worse than that of the SI-based method, the PAPR reduction achieved by the transparent method is comparable.

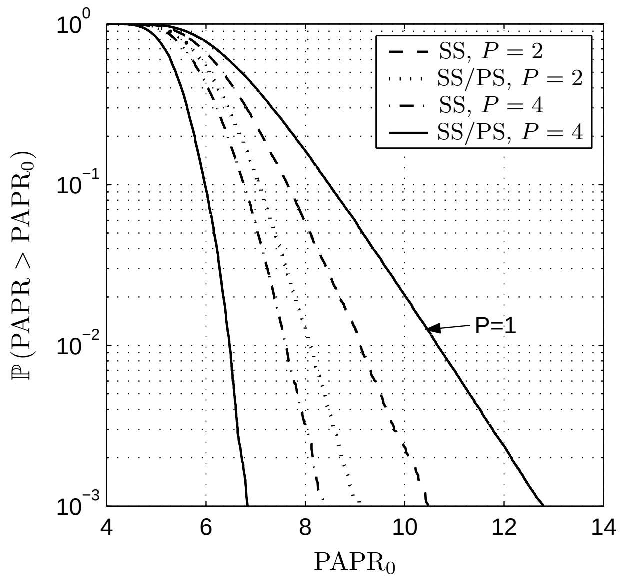

Fig. 6. CCDF of the PAPR for the transparant SS and combined SS/PS method for the PR subcarrier grouping scheme, for $N_\mathrm{t} = 2$ and $M = 10$. PAPR is calculated for the entire 10 OFDM symbol packet.

VIII. Conclusions and Discussion

Spatial shifting (SS) is proposed to reduce the peak-to-average power ratio (PAPR) of the time domain signals in multiple antenna OFDM systems. Hereto the extra degree of freedom provided by the MIMO system is used to reshuffle groups of subcarriers such that the resulting TX signals attain a low PAPR.

The numerical results in this paper support the conclusion that significant gain in PAPR reduction can be achieved by combining SS with phase shifting (PS) of the subcarrier groups. Moreover, the paper proposes the application of the SS and the combined SS/PS method in a transparant mode, where no extra overhead is introduced by the transmission of side-information.

It can, furthermore, be concluded from the presented results that the performance of SS, differently from PS, does not depend on the chosen subcarrier grouping. Therefore, the grouping scheme can be chosen that minimizes the computational complexity of the implementation of SS.

It is noted that the PAPR reduction approaches were proposed and evaluated for discrete-time signals. Therefore, the presented results will only be directly applicable to continuous-time signals when ideal bandpass filtering of the signals can be assumed. When this is not the case, the achieved PAPR reduction might be lower and suboptimal [8], [13]. To achieve effective PAPR reduction for these cases, the presented algorithms could be modified to apply oversampling in the algorithm, i.e., by zero-padding of the signal $\mathbf{S}_{m,n_\mathrm{t}}^{(p)}$ and by increasing the size of the DFT matrix in (6).

Further investigation is necessary to reduce the computational complexity of the proposed methods. Hereto, suboptimal techniques can be investigated as previously proposed for conventional OFDM in e.g. [7].

Another subject of further research is PAPR reduction in MIMO OFDM systems, which already exploit the extra degree of freedom by transmit processing, like e.g. transmit beamforming. In these kind of systems the systems performance does, differently from space division multiplexing [14], depend on the specific ordering of the TX streams. Hereto the straightforward application of SS for these kind of systems is not recommended.

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