Framed Random Access for Reliable Light Source Identification in Smart Lighting Systems

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Light emitting diodes (LEDs) will form the basis for smart lighting systems of the future. To enable intuitive user interaction with a large number of these light sources, identification using visible light communication (VLC) has been proposed [1,2]. This paper proposes the use of asynchronous framed random access for this identification. The system performance is analyzed in terms of identification probability, both analytically and numerically. To maximize this performance, we propose several approaches. From simulation results we conclude that applying these methods yields efficient and accurate identification.

Introduction

The use of LEDs as basis for lighting systems attracts increasing attention [3]. It is foreseen that LED lighting system will be formed by a large number of distributed LEDs of various colors. By switching and dimming these LEDs, one can adaptively set the light configuration and create an nearly infinite variety of lighting patterns / atmospheres. The large number of degrees of freedom in control might form a challenge for end-users. Therefore, the authors of [1,2] propose a system where the light sources embed invisible identifiers in their light output, which are received by a light sensor-equipped remote control. Consequently, the system can identify the contributions of the locally relevant light sources, and control their light output based on user requests. This control can be done via another link, e.g. based on RF wireless communications. The communication address of the light source for this second link might serve as identifier on the VLC link.

These previously proposed solutions, however, require the set of (synchronous CDM or asynchronous FDM) identifiers to be assigned in the system before operation. This might be impractical for some applications, hence, we propose a new approach based on *random access* VLC in which light sources transmit a factory embedded address, e.g. their MAC address. The considered system consists of an asynchronous unidirectional light link, where the challenge is to identify all light sources within minimum time. To deal with collisions we consider a framed ALOHA approach [4], where every light source repeatedly transmits its identifiers once in every time frame. Due to the unidirectional link, we more specifically focus on framed ALOHA without acknowledgements (ACK).

The ALOHA-based protocols have been extensively analyzed in literature, see e.g. [5, 6], however, these contributions mainly focussed on system throughput and stability. It is noted that maximum throughput does not directly translate in maximum identification performance, since for the latter we have to receive packets from *all* light sources. ALOHA has also been considered for identification in RFID systems [7, 8], however, these contributions were limited to slot-synchronized systems and systems applying ACK.

To this end, this paper extends previous literature by considering the identification performance of framed ALOHA without ACK for the continuous, non-slotted, scenario. We provide analytical and numerical performance results, for the considered lighting system scenario, as well as algorithms for system optimization.

System Model

We consider light source identification based on a framed ALOHA VLC transmission scheme for a system consisting of N LED light sources. Each LED transmits its identifier (ID) sequence individually without knowing the transmission states of others. Furthermore, since there is no feedback path, the remote control cannot send an ACK to the LEDs to confirm successful identification. Thus, the LED has no knowledge about its own identification state. In order to guarantee that the ID of each LED is at least once successfully received, multiple frames are used. In each frame, each LED sends its ID packet once.

Figure 1 illustrates the frame structure for $N = 4$ LEDs, each sending an ID packet (indicated by blocks here) of length L sec, for frame length $T = mL$ sec, where m denotes the normalized frame length. In the first frame, a collision between the packets of LED 2 and 4 occurs, but LED 1 and 3 can be successfully identified. Whereas, in the second frame, a collision occurs for LED 1 and 4, and LED 2 is newly identified.

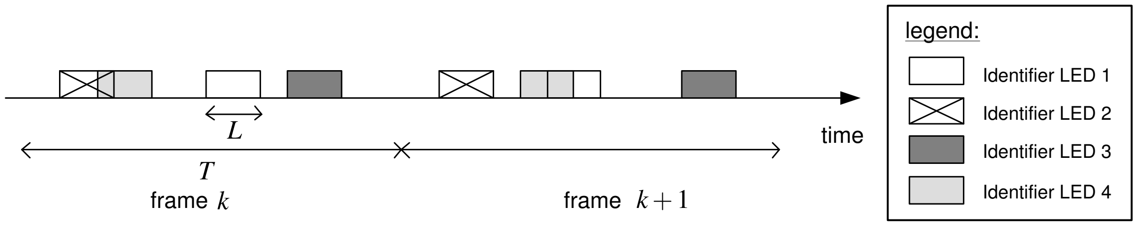


Figure 1: Frame structure for the random access scheme with 4 LED light sources.

Thus, after two frames, three LEDs are detected. After M frames, the identification process is stopped, i.e. after $T_{\mathrm{tot}} = MT = mML$ sec.
We propose to locate the ID sequence of a certain LED uniformly within in a frame, to avoid recurring collisions. The ID packets, however, cannot cross the boundaries of two successive frames. Hence the probability density function of the starting point x of the ID packet is given by

$$p(x) = \begin{cases} \frac{1}{T-L} & 0 \leq x < T-L \\ 0 & T-L \leq x < T \end{cases} . \tag{1}$$

The probability that the identification packets of all N LEDs are successfully received, i.e. without collisions, within the detection time T_{tot} will be referred to as *detection probability* and is denoted by $P_{\mathrm{d},M}$. The identification scheme should either maximize $P_{\mathrm{d},M}$ within a required detection time T_{tot}, or minimize T_{tot} for a given $P_{\mathrm{d},M}$.

Performance evaluation

Due to the asynchronous nature of the problem, the LEDs will not have a common notion of time. Hence, the frames of the different LEDs will not be aligned. To facilitate analysis, however, we will consider aligned frames anyway. Later in this section, we verify that the impact of this assumption is minor. A packet is defined to be correctly received when no collision occurs with the packets of any of the $N-1$ other LEDs. A collision is defined as a (partial) overlap between the two packets. Hence, capture effects are considered negligible in this analysis.
The probability that a labelled LED can be identified within a single frame, P_1, is derived in the appendix, yielding

$$P_1 = I(T \geq 2L)\frac{2}{N}\left(\frac{T-2L}{T-L}\right)^N + I(T \geq 3L)\left(1-\frac{2}{N}\right)\left(\frac{T-3L}{T-L}\right)^N , \tag{2}$$

where $I(a \geq b)$ equals 1 for $a \geq b$ and 0 otherwise. The probability that all N LEDs can be identified within a single frame, i.e. $M = 1$, is shown in the appendix to equal

$$P_{\mathrm{d},1} = I(T \geq NL)\left(\frac{T-NL}{T-L}\right)^N . \tag{3}$$

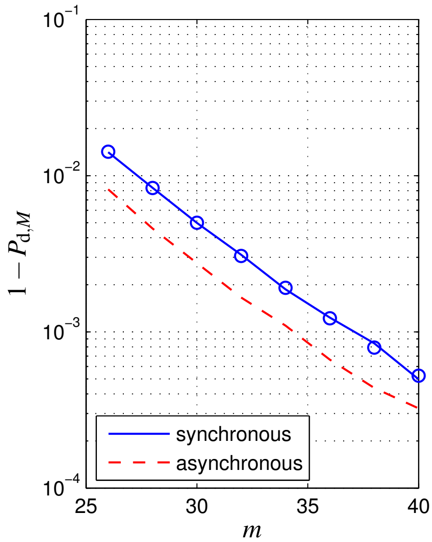
Let us denote the event that the nth LED can be identified after M frames as E_n. Because the identification processes are identical for all frames, we find $P(E_n) = 1-(1-P_1)^M$. Using this, the exact expression of $P_{\mathrm{d},M}$ is given by

$$P_{\mathrm{d},M} = P(E_1) \cdot P(E_2|E_1) \cdot P(E_3|E_2,E_1) \cdot \ \ldots \ \cdot P(E_N|E_{N-1},\ldots,E_1). \tag{4}$$

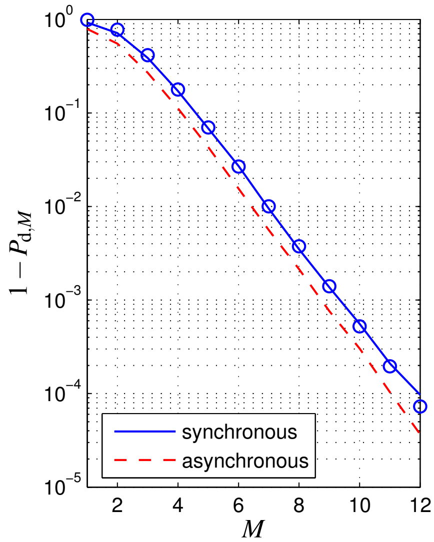
Since in the considered scenario N, M and m are not small, the identification states of the different LEDs can be considered to be approximately independent. Thus, $P(E_n|E_{n-1},\ldots,E_1) \approx P(E_n) \forall n = 2,\ldots,N$. Hence, $P_{\mathrm{d},M}$ can be well approximated as

$$P_{\mathrm{d},M} \approx (P(E_n))^N = \left(1-(1-P_1)^M\right)^N . \tag{5}$$

Numerical results for $P_{\mathrm{d},M}$ are given by the top curves in Fig. 2 ("synchronous"), which compares the analytical results of (5) with results from Monte Carlo simulations for N = 10 LEDs and M = 10 frames.



(a) Results for N=10 and M=10



(b) Results for N=10 and m=40

Figure 2: Analytical (markers) and simulation (lines) performance results for synchronous (solid) and asynchronous frames (dashed).

One observes a nice match between the results, revealing the approximation in (5) is justified. Note that in practice N would be even higher than 10, making the approximation even better.
The figure, moreover, illustrates the simulation results for asynchronous frames, in which the beginning of the frames is not aligned between the different LEDs. From these results, and from analytical results not presented here, we conclude that the detection probability for the scheme with asynchronous frames is lower bounded by that for synchronous frames. The synchronous case can thus be considered a *worst case*, which can be explained due to the lower interference distribution near frame edges due to the allocation function in (1).

System optimization

The optimal system is now defined as the system that maximizes the detection probability for a given detection time T_{tot}. Herein the freedom exists to create either many short frames or several long frames, i.e. a tradeoff between m and M. When the frame length is increased, the detection probability within a frame will also increase. However, consequently also M decreases, which will yield a smaller $P_{\text{d},M}$ in the end. Similarly, a too small frame length is also not good for the performance. Hence, optimal values of m and M exist and these will be derived below.
To this end, let us rewrite (2) as a function of m. Using $m = T/L$ we find that

$$P_1(m) = \frac{2}{N}\left(\frac{m-2}{m-1}\right)^N + \left(1-\frac{2}{N}\right)\left(\frac{m-3}{m-1}\right)^N, \tag{6}$$

under the condition that $m \geq 3$.
Combining (6) with (5), we find that

$$P_{\text{d},M} = \left(1-(1-P_1(m))^{T_{\text{tot}}/m}\right)^N. \tag{7}$$

For a system with constant N and a required T_{tot}, the probability $P_{\text{d},M}$ is only a function of the normalized frame length m. Hence m should be optimized, and the optimal value is denoted as m_{o}. Subsequently, we can find the corresponding number of optimal frames as $M_{\text{o}} = T_{\text{tot}}/m_{\text{o}}$. The optimal m can now be found as

$$m_{\text{o}} = \arg\max_m \{P_{\text{d},M}(m)\} = \arg\min_m \left\{(1-P_1(m))^{T_{\text{tot}}/m}\right\} = \arg\min_m \left\{\frac{\ln(1-P_1(m))}{m}\right\}. \tag{8}$$

Hence, T_{tot} does not influence m_{o}. It indicates that for determining m_{o}, we only need to consider the number of LEDs involved and do not need to know the detection time T_{tot}.
From the numerical results for (8) we find that the optimum frame length is well approximated by

$$m_{\mathrm{o}} = 0.78 + 2.89N. \tag{9}$$

It is noted that this optimum frame length is different from the frame length m^* that maximizes the throughput $NT_{\mathrm{tot}}P_1(m)/m$, for which it can be shown that $m^* \approx 2N + 4 - 2\sqrt{e}$.
Alternatively, one can derive the optimal settings for a system minimizing the required T_{tot} to achieve a certain $P_{\mathrm{d},M}$. It can be easily shown that the optimal settings for this scenario are also given by (9).

Number of sources estimation

The problem with the use of (9) during system operation is that it requires knowledge about the number of LEDs N, which typically will not be available before identification. Therefore we propose a method that estimates the number of sources during the identification process. If we consider the case of a required detection performance $P_{\mathrm{d},M}$, this can be used to minimize the total detection time T_{tot}. The proposed procedure is summarized in Fig. 3.

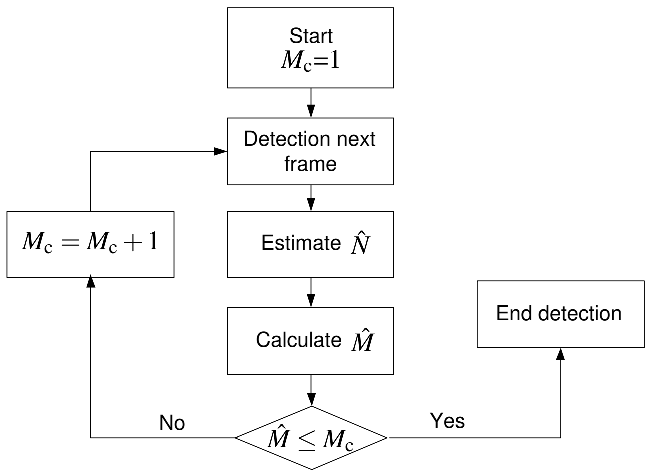


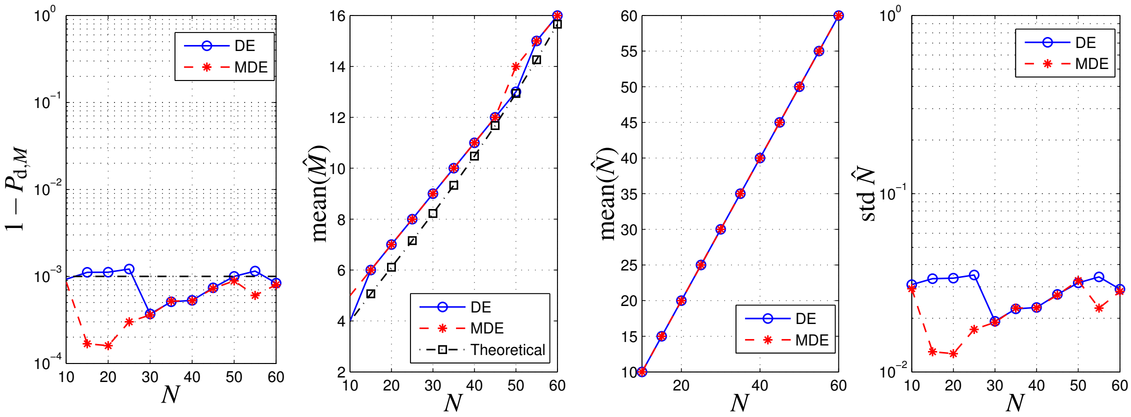
Figure 3: Procedure for the estimation of the number of required frames $\hat{M}$.

First the counter for the number of frames M_{c} is initialized. After the reception of every frame, an updated estimate of the number of sources $\hat{N}$ in the system is made. Here we assume a fixed m is used, which is optimized for a typical or maximum N. Based on this, the number of required total frames is estimated as

$$\hat{M} = \left\lceil \frac{\ln\left(1 - P_{\mathrm{d},M}^{1/\hat{N}}\right)}{\ln\left(1 - P_1\right)} + \alpha \right\rceil , \tag{10}$$

where $\alpha = 0$ in our first approach, named direct estimation (DE), and (10) directly follows from (7). If $\hat{M}$ is smaller or equal to the number of already acquired frames M_{c}, the identification process is stopped. It is stopped since the algorithm indicates that the number of acquired frames is adequate to meet the required detection performance $P_{\mathrm{d},M}$. If this is not the case, the next frame is detected.
Key for this detection scheme is the estimation algorithm for the number of LEDs. We propose a simple approach in which N is estimated based on the number of individual LEDs identified so far, denoted as N_{i}. The principle behind this is that the majority of the LEDs are identified in the first few frames. Most of the frames are required to identify the last few LEDs. Hence, one just needs to track N_{i} and the estimate of the number of LEDs then simply is $\hat{N} = N_{\mathrm{i}}$. Subsequently, $\hat{M}$ follows from (10).
It is clear from the above that the DE algorithm always yields a $\hat{N}$ which is smaller than or equal to the actual N. Due to this underestimation it might in some cases result in a too low number of frames to achieve the required detection probability. To overcome this, a modified version of the DE algorithm, named *modified direct estimation* (MDE), is proposed here. In the MDE algorithm α in (10) is a positive non-zero value,

to avoid underestimation of the number of frames. Generally, one would select $\alpha \in [0,1)$. A large α can guarantee the required $P_{\rm d}$ to be met, but it may lead to a long detection time. On the other hand, a small α may not always result in the required $P_{\rm d}$.
Numerical results were obtained for the algorithms using Monte Carlo simulations, to understand their performance. The results are presented for a varying number of light sources, $m = 174$ (optimum for 60 sources) and a required detection performance of $P_{{\rm d},M} = 1 - 10^{-3}$. For the MDE algorithm we used $\alpha = 0.15$. The results are depicted in Fig. 4.



(a) (b) (c) (d)

Figure 4: Simulation results for the DE and MDE algorithms as function of N: (a) average fail probability $1 - P_{{\rm d},M}$, (b) mean of estimated M, (c) mean of estimated N, (d) standard deviation of estimated N.

We can conclude from Fig. 4(a) that both estimation approaches perform well. The MDE algorithm can always guarantee the required detection probability. The DE algorithm, however, sometimes cannot, although the difference with the required 10^{-3} is small. Figure 4(b) illustrates that the MDE algorithm is more conservative than the DE algorithm, i.e. it always overestimates the the required number of frames as shown by the curve “theoretical”. It is noted, however, that the increase in required detection time is limited for MDE as compared to DE.
The accuracy in estimation of N is illustrated from Fig. 4(c), which shows the mean of $\hat{N}$ at the end of the last iteration. It is clear that the mean of $\hat{N}$ follows N. We can consequently conclude that the estimation seems to be unbiased. For a better understanding of the estimation performance, the standard deviation (std) of $\hat{N}$ is depicted in Fig. 4(d). It is observed that the standard deviation values are relatively small compared to the values of N. It is interesting to note that the trend of the curve for the std is similar to that of $1 - P_{{\rm d},M}$. An intuitive explanation is that a smaller std leads to a better estimation of $\hat{M}$, hence the probability that $\hat{M}$ is smaller than the theoretical optimum is reduced. Consequently, $1 - P_{{\rm d},M}$ will be smaller.

Conclusions

The use of framed ALOHA without ACK has been proposed in this paper for source identification in general, and LED identification in intelligent lighting system more specifically. The probability of successful identification of all LEDs, $P_{{\rm d},M}$, was used as the optimization criterium for the system. We derived an approximation expression for this probably which closely lowerbounds the exact value. Simulation results confirmed these findings.
To maximize system performance for practical implementations, an optimization of the system was performed, yielding an optimal tradeoff between the frame length and number of frames. It was found that the optimal frame length is approximately 2.89 times the number of sources. Also, two estimation approaches for the number of required frames during acquisition are proposed. Numerical results showed that these can be used to efficiently identify all LEDs, while guaranteeing the required performance.

Appendix

Derivation of $P_{\mathbf{d},1}$

Let us denote the starting time of 1st, 2nd, …, Nth LED ID transmission as $t_1, t_2, \ldots, t_N$. The joint pdf of the t_i for $i = 1, \ldots, N$ is given by $p_c(t_1, \ldots, t_N) = 1/(T-L)^N$. We order the t_i and let $(s_1, \ldots, s_N) = \text{sort}(t_1, \ldots, t_N)$, so that $s_1 < s_2 < \cdots < s_N$. The event that any two t_i are equal has zero measure. Denote S_N as the set of all permutations of $1, 2, \ldots, N$. Then the joint pdf of $s_1, s_2, \ldots, s_N$ is given by

$$
\begin{aligned}
p_s(s_1, \ldots, s_N) &= I(0 \le s_1 < \cdots < s_N < T-L) \sum_{\pi \in S_N} p_c(s_{\pi(1)}, \ldots, s_{\pi(N)}) \\
&= I(0 \le s_1 < \cdots < s_N < T-L) \frac{N!}{(T-L)^N}. \qquad (11)
\end{aligned}
$$

Then $P_{\mathrm{d},1}$ is given by

$$
\begin{aligned}
P_{\mathrm{d},1} &= \int_0^{T-L} \cdots \int_0^{T-L} I(\forall_m \forall_{n \neq m} |t_m - t_n| \ge L) p_c(t_1, \ldots, t_N)\, \mathrm{d}t_1 \ldots \mathrm{d}t_N \\
&= \int_0^{T-L} \cdots \int_0^{T-L} I(\forall_{m \in \{1,\ldots,N-1\}} s_{m+1} - s_m \ge L) \frac{N!}{(T-L)^N}\, \mathrm{d}s_1 \ldots \mathrm{d}s_N \\
&= \frac{N! I(T \ge NL)}{(T-L)^N} \int_0^{T-NL} \left[\int_{s_1+L}^{T-(N-1)L} \left[\int_{s_2+L}^{T-(N-2)L} \left[\cdots \left[\int_{s_{N-1}+L}^{T-L} \mathrm{d}s_N \right] \cdots \right] \mathrm{d}s_3 \right] \mathrm{d}s_2 \right] \mathrm{d}s_1 \\
&= \frac{N! I(T \ge NL)}{(T-L)^N} \int_0^{T-NL} \left[\int_{y_1}^{T-NL} \left[\cdots \left[\int_{y_{N-1}}^{T-NL} \mathrm{d}y_N \right] \cdots \right] \mathrm{d}y_2 \right] \mathrm{d}y_1 \\
&= I(T \ge NL) \left(\frac{T-NL}{T-L} \right)^N, \qquad (12)
\end{aligned}
$$

where in the last step we used $s_i = (i-1)L + y_i$ for $i = 1, \ldots, N$.

Derivation of P_1

Without loss of generality, we consider the first LED as the labeled source. Using the above, P_1 can be expressed as

$$
\begin{aligned}
P_1 &= \int_0^{T-L} \cdots \int_0^{T-L} I(\forall_{m>1} |t_m - t_1| \ge L) p_c(t_1, \ldots, t_N)\, \mathrm{d}t_1 \ldots \mathrm{d}t_N \\
&= \frac{1}{(T-L)^N} \int_0^{T-L} \left((t_1 - L) I(t_1 - L \ge 0) + (T - 2L - t_1) I(T - 2L - t_1 \ge 0) \right)^{N-1} \mathrm{d}t_1 \\
&= I(2L \le T < 3L) \frac{2}{N} \left(\frac{T-2L}{T-L} \right)^N + I(T - 3L \ge 0) \left[\frac{2}{N} \left(\left(\frac{T-2L}{T-L} \right)^N - \left(\frac{T-3L}{T-L} \right)^N \right) + \left(\frac{T-3L}{T-L} \right)^N \right] \\
&= I(T - 2L \ge 0) \frac{2}{N} \left(\frac{T-2L}{T-L} \right)^N + I(T - 3L \ge 0) \left(1 - \frac{2}{N} \right) \left(\frac{T-3L}{T-L} \right)^N. \qquad (13)
\end{aligned}
$$

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