Digital Compensation of Amplifier Nonlinearities in the Receiver of a Wireless System

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***Abstract* — In order to enable efficient implementation of wireless systems with high signal dynamics, this paper proposes an approach to estimate and correct for the amplitude and phase distortion caused by nonlinear power amplifiers. The approach comprises a preamble design, an estimation method and a digital compensation algorithm. Correction for the nonlinearities is applied in the receiver, which requires no additional hardware. Performance evaluation of a 60 GHz OFDM-based system shows that the proposed approach can successfully be applied with minimal performance degradation compared to non-impaired systems.**

***Index Terms* — Amplifiers, Nonlinearities, Compensation, Digital signal processing, mm-wave communications.**

I. Introduction

The definition of the most suitable modulation format for 60 GHz wireless personal area network (WPAN) systems is currently under discussion in several standardization bodies. Several proposals are based on orthogonal frequency division multiplexing (OFDM), which has the advantage of high achievable data rate and robustness to multipath fading. However, the high spectral efficiency of OFDM comes at the cost of an increase in signal dynamics. This proves to be an important performance limiting factor when efficient, i.e., intrinsically nonlinear, amplifiers are used in system implementation. To minimize the impact of these nonlinearities, significant power back-offs are required when feeding the signal to the transmit amplifier, reducing its operational range and efficiency.

Since operational range is a key issue in 60 GHz systems, this gives the advantage to constant envelop modulations such as continuous phase modulation (CPM), minimum shift keying (MSK) and OFDM phase modulation (OFDM-PM) for application in 60 GHz WPAN systems. This is because these modulation formats allow for a power-limited power amplifier to be operated near its saturation level, as such, maximizing the power efficiency. In this paper we propose a set of techniques which also allow for efficient operation of transmit amplifiers for modulation formats with high signal dynamics, such as OFDM. Hence, this might change the discussion about the most suitable modulation for 60 GHz WPAN.

To reduce the impact of nonlinearities, several digital signal processing based techniques have been proposed to reduce the peak-to-average power ratio (PAPR) of the transmit signals for wireless systems, see e.g. [1] and [2] for CDMA and OFDM systems, respectively. Moreover, techniques have been proposed based on digital predistortion in amongst others [3] and [4], where the latter also treats memory effects. However, due to the additional processing required for PAPR reduction and predistortion, they are generally not applicable for implementation at the mobile device side in uplink transmissions. Moreover, predistortion methods require power consuming broadband A/D converters, which annihilate the achieved efficiency improvement. Also, the introduced delays due to feedback loops in the predistortion might not be acceptable in practical implementations.

Therefore, this paper proposes to employ the additionally required processing for reduction of the influence of nonlinearities in the receiver base station/access point by means of postdistortion. We propose a preamble design, which enables separate estimation of the channel state information (CSI) and the nonlinearities. Compensation for the influence of the transmitter nonlinearities is achieved before channel equalization in the receiver and, as such, it reduces the complexity of the total implementation. Moreover, we, differently from the majority of previous literature, treat the combined influence of both amplitude (AM/AM) and phase (AM/PM) distortion due to nonlinearities.

This paper is organized as follows. First Section II provides an overview of nonlinear power amplifier models and shows that these models can be developed into polynomial series expansions. It also introduces orthogonal polynomial bases to enable reliable estimation of, and correction for, the nonlinearities. Section III, subsequently, introduces the system model of a wireless system experiencing nonlinearities. A method to accurately estimate the nonlinearities in a polynomial basis is the proposed in Section IV. Following this, a low complexity compensation algorithm is presented in Section V. Simulation results revealing the performance of this

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approach in a 60 GHz OFDM system are reported in Section VI. Finally, we conclude in Section VII.

II. Nonlinearity Models and approximations

A. Nonlinearity modeling

The response of broadband power amplifiers can have precarious memory effects. For wireless devices with limited bandwidth compared to the applied carrier frequency, these effects can be effectively minimized by careful design [4]. Moreover, these amplifier nonlinearities with memory effect can generally be reduced to a Wiener or Hammerstein model [5], i.e., the concatenation of a linear filter and nonlinear memoryless function. For the Hammerstein model the linear filter function can effectively be regarded as being part of the propagation channel. Consequently, we will in the following assume the memory effects to be negligible.

The influence of a memoryless nonlinearity $f(\cdot)$ can be decomposed into an amplitude distortion $g_G(\cdot)$ and a phase distortion $g_\varphi(\cdot)$, which are both functions of the amplitude of the input signal. The complex signal $s_1(t)$ at the output of the nonlinearity can, consequently, be written as

$$\begin{aligned} s_1(t) &= f(s(t)) = g(|s(t)|)s(t) \\ &= g_G(|s(t)|)\exp(jg_\varphi(|s(t)|))s(t), \end{aligned} \tag{1}$$

where $g(\cdot)$ denotes the nonlinear distortion function, $|\cdot|$ denotes the absolute value and $s(t)$ is the complex signal input to the nonlinearity.

In the literature two models of nonlinearities are widely used for wireless systems, i.e., [7] and [8]. Both were originally proposed for power amplifiers.

- The solid-state amplifier (SSA) model, as proposed by Rapp in [7], for which the nonlinearity is described by

$$g(|s(t)|) = \left(1 + \left(\frac{|s(t)|}{A_{\max}}\right)^{2p}\right)^{-\frac{1}{2p}}, \tag{2}$$

where $A_{\max}$ is the output level at saturation and p determines the smoothness of the transfer and is a positive integer.

- The traveling wave tube (TWT) amplifier model, as proposed by Saleh in [8], for which the nonlinearity is described by

$$g(|s(t)|) = \frac{\alpha_G}{1+\beta_G|s(t)|^2}\exp\left(j\frac{\alpha_\varphi|s(t)|^2}{1+\beta_\varphi|s(t)|^2}\right), \tag{3}$$

where α_G, β_G are the parameters describing the amplitude nonlinearity and α_φ, β_φ are the phase displacement parameters.

It has been shown previously that by a Taylor series expansion around zero, these models can be brought back to odd-order polynomials [9]. If we define $x=|s(t)|$ to be the amplitude of the transmit signal, we can rewrite the distortion function as

$$\begin{aligned} g(x) = &\left(1 + A_3x^2 + A_5x^4 + \ldots + A_{2n+1}x^{2n} + o\left(x^{2(n+1)}\right)\right)\cdot \\ &\exp\left(j\left(\Phi_3x^3 + \Phi_5x^5 + \ldots + \Phi_{2n+1}x^{2n+1} + o\left(x^{2n+3}\right)\right)\right), \end{aligned} \tag{4}$$

where A_n and Φ_n denote the nth order polynomial coefficients for $g_G(\cdot)$ and $g_\varphi(\cdot)$, i.e., the AM/AM and AM/PM nonlinear function, respectively. We note that the coefficients of the AM/AM characteristic can be related to the amplitude of the power amplifier input interception points, the common measures to characterize nonlinear amplifiers. For the third and fifth order coefficients this yields

$$A_3 = \frac{4}{3A_{\mathrm{IIP3}}^{\;2}} \quad \text{and} \quad A_5 = \frac{8}{5A_{\mathrm{IIP5}}^{\;4}}, \tag{5}$$

respectively. Here A_{IIP3} and A_{IIP5} denote the third and fifth order input interception points, respectively.

B. Orthonormal polynomial basis

In fact, the previous polynomial approximation corresponds to the projection of $g_G(\cdot)$ and $g_\varphi(\cdot)$ onto the canonical basis

$$\{e_0, e_1, e_2, e_3, e_4, e_5, \ldots\} = \{1, x, x^2, x^3, x^4, x^5, \ldots\}. \tag{6}$$

It is well-known, however, that identification of non-linearities is difficult in this basis, since it will easily incur numerical errors [10].

Therefore, we will derive a more suitable orthonormal polynomial basis, using the Gram-Schmidt orthonormalisation method. For this orthonormalisation, we will apply the inner product $\langle u,v\rangle = E[u(t)v(t)]_{t\in[0,T]}$ over the observation period T. For a system sampled with a sample time $T_s = T/N$, the inner product can be rewritten as $\langle u,v\rangle = \sum_{k=0}^{N} u(k)v(k)$. This bilinear form is symmetric and positive-definite.

Now, let us define $\chi_n = E[x^n]$ to be the nth moment of the transmitted signal amplitude. The iterative steps of the Gram-Schmidt orthonormalization process result into the terms of the orthonormal polynomial basis $\{p_0(x), p_1(x), p_2(x), \ldots\}$, which are given by

$$p_{i+1} = \frac{e_{i+1} - \sum_{k=0}^{i}\langle e_{i+1}, p_k\rangle p_k}{\left\| e_{i+1} - \sum_{k=0}^{i}\langle e_{i+1}, p_k\rangle p_k \right\|}, \tag{7}$$

where e_i is the ith element of the canonical basis as defined in (6). The first four orders of the developed orthonormal polynomial basis, as function of the moment χ_n, are given by

$$\begin{aligned} &p_0(x) = 1, && p_1(x) = \frac{x}{\sqrt{\chi_2}}, \\ &p_2(x) = \frac{x^2 - \chi_2}{\sqrt{\chi_4 - \chi_2^{\;2}}}, \text{ and} && p_3(x) = \frac{\chi_2 x^3 - \chi_4 x}{\sqrt{\chi_2^{\;2}\chi_6 - \chi_4^{\;2}\chi_2}}. \end{aligned} \tag{8}$$

We note that according to (4) the orthonormalization process can be reduced to the odd-terms of the canonical basis. The interest of the whole polynomial basis is, however, stressed in [10]. The identification of the nonlinearity parameters, as we will present in Section IV, is facilitated by the use of this orthonormal basis.

C. Polynomial interpolation

We also consider the use of interpolation polynomials, such as Lagrange or Newton, since they are good candidates for low complexity approximation of continuous nonlinear functions.

Hence, we propose their application in Section V for the nonlinearity compensation.

Let $\{a_i,b_i\}$ be the points for the corresponding interpolation polynomial $L\{a,b\}(x)$, where generally $b = g(a)$. The first point of the polynomial, i.e., $\{a_0,b_0\}$ should be zero, since the amplifiers exhibit a linear behavior for low input amplitudes. The quadratic error in the approximation of the distortion using this polynomial can then be expressed as:

$$\varepsilon = \int_0^{A_{\max}} (L\{a,b\}(x) - g(x))^2 P_{|s(t)|}(x)dx, \quad (9)$$

where $P_{|s(t)|}(x)$ is the probability density function of the envelope of the transmit signal. For an OFDM signal $P_{|s(t)|}(x)$ is well approximated by a Rayleigh distribution.

Although one could choose the abscissa points a_i to be linearly distributed, it seems more optimal to put more points close to zero. This is especially true for systems applying signals with a high dynamic range, such as an OFDM system with a high number of carriers, since the high input values, near $A_{\max}$, have a low probability of occurrence.

Let us now define γ to be the linear distribution of the abscissa points, with a minimum and maximum value of 0 and 1, respectively. When we then focus on powers of γ as possible distributions of the abscissa points, numerical evaluation of the quadratic error in the approximation, as defined in (9), can be used to find the optimal abscissa points for the combination of input signal and nonlinearity. This is illustrated by Table 1 for a TWT amplifier with α_G=1, β_G=0.15, α_φ=0.07 and β_φ=0.1 and an OFDM signal with different variances for the amplitude of the input signal.

Variance of the input signals envelope distribution	0.05	0.1	0.15	0.2
Best abscissa points	γ^7	γ^4	γ^2	$\gamma^{1.5}$

Table 1: Optimal interpolation abscissa points for different distributions of the amplitude of the input signal.

For unknown input signal distribution and/or power amplifier attributes, the "squared linear basis" with M points

$$a = \left(0, \left(\frac{1}{M}\right)^2, \ldots, \left(\frac{M-1}{M}\right)^2\right) A_{\max} \text{ and } b = g(a), \quad (10)$$

leads to a good approximation for a wide range of input signal distributions and amplifier characteristics.

We note that another possible method could be the use of Chebyshev nodes, which are the roots of the Chebyshev polynomial of the first kind, since they can also be used to minimize the problem of unwanted oscillations in the interpolation polynomial process, also known as Runge's phenomenon [11].

III. SYSTEM MODEL

Let us now consider a baseband equivalent model for our transmission scheme, as schematically depicted in Fig. 1.

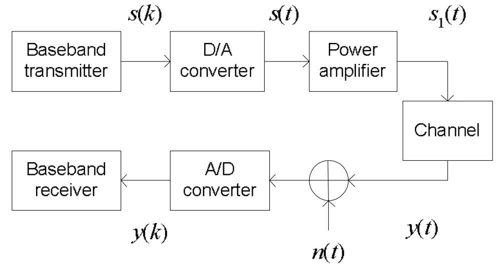

Fig. 1: Schematic model of the regarded wireless system.

The received baseband signal after transmission through the wireless channel can be written as

$$y(t) = g(|s(t)|) \cdot s(t) * h(t) + n(t), \quad (11)$$

where $h(t)$ and $n(t)$ denote the complex channel impulse response and additive white Gaussian noise, respectively. Here * denotes convolution. We can conclude from (11) that the received signal is influenced by a channel transfer function, a nonlinear function and a random noise process.

In the following we will focus on a specific system, i.e., an (uncoded) OFDM system, to make our approach more intelligible. It is noted, however, that the proposed techniques are more generally applicable. We regard a system which transmits a preamble with training sequence q in front of the actual data packet. Also, a cyclic prefix is added to every OFDM symbol in the time domain to create robustness against inter-symbol interference.

The processing we propose in order to compensate for both the nonlinearities and the channel in such an OFDM system is summarized in Fig. 2 and will be further detailed in Sections IV and V. It consists of deriving an estimate of the (wireless) channel and the nonlinearity and subsequently correcting for both. For optimum compensation of both effects, compensation for the nonlinearities should theoretically occur after the channel equalization. However, to yield an implementation with limited complexity, nonlinearity compensation in this work will be done before the FFT operation. In Section VI we show this yields a good bit-error rate (BER) performance.

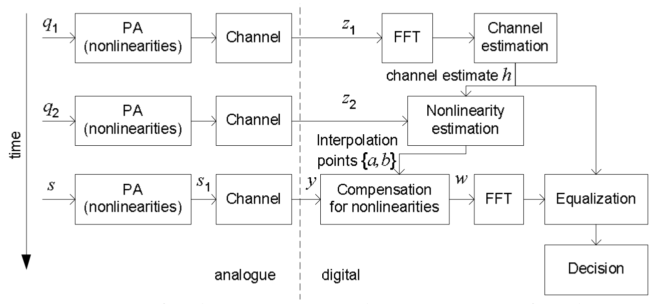

Fig. 2: Strategy for the estimation and compensation of nonlinearities in the receiver of an OFDM system.

IV. ESTIMATION OF THE NONLINEARITIES

A. Preamble design

To enable the separate estimation of the nonlinearities and the channel response, we propose a new preamble format q consisting of two sequences:

- A constant modulus training sequence q_1 for (linear) channel estimation. q_1 should be frequency white to enable accurate estimation of frequency selective channels.
- A high PAPR training sequence q_2 for nonlinearity estimation. q_2 should cover the same dynamic range as the data signal and have the same mean power.

B. Linear channel estimation

The received signal corresponding to the transmitted training sequence q_1 is given by

$$\begin{aligned} z_1(t) &= g(|q_1(t)|)q_1(t) * h(t) + n(t) \\ &= q_1(t) * Q_1 h(t) + n(t), \end{aligned} \tag{12}$$

where $Q_1 = g(|q_1(t)|)$ is time-independent due to the constant modulus property of q_1. An estimate of the channel state information is derived from the received preamble symbol z_1, using a classical least-squares or minimum mean squared error estimation algorithm [12]. Note that a scaling of Q_1 will be present in this estimate, which can be simply removed after estimation of the nonlinearity.

C. Nonlinearity estimation

With the knowledge of the moments of the training sequence q_2, a dedicated orthonormal basis is computed using (7), to prevent numerical errors in the identification of the non-linearities. The nonlinear function can be expressed in this polynomial basis $\{p_0(x), p_1(x), p_2(x), ...\}$ as

$$g(x) = \sum_{k=1}^{\infty} g_k p_k(x) \exp\left(j\left(\sum_{k=1}^{\infty} \varphi_k p_k(x) \right)\right). \tag{13}$$

A suitable approximation of $g(\cdot)$ is the Kth order series

$$\hat{g}(x) = \sum_{k=1}^{K} g_k p_k(x) \exp\left(j\left(\sum_{k=1}^{K} \varphi_k p_k(x) \right)\right). \tag{14}$$

The projection of the received signal z_2, corresponding to the transmitted training sequence q_2, onto the basis of (14) gives a reliable approximation of the AM/AM g_k and AM/PM φ_k nonlinearity characteristics.

Alternatively, one could use a similar method to directly estimate the inverse of the nonlinearity, as was illustrated by the authors in [13] for a MIMO OFDM system.

D. On approximation accuracy

The accuracy of the approximated function depends on:

- The series length K compared to the effective order of the power amplifier nonlinearity.
- The length of the training sequence relative to the signal-to-noise ratio.

From our experience we conclude that a fifth order polynomial can cover a wide range of power amplifier classes and technologies. We apply (10) to be the basis for the polynomial interpolation with $b = \hat{g}(a)$.

V. COMPENSATION FOR THE NONLINEARITIES

Postdistortion would be most straightforwardly carried out by applying the estimate of the inverse nonlinear function to the received signal $y(t)$. The inversion of the estimated nonlinearity function $\hat{g}$ is, however, often difficult to calculate. However, the inversion of abscises and ordinates of the interpolation points is simple. As previously introduced in Section III.C, the interpolation polynomial $L\{a,b\}$ can give an accurate approximation of g. Hence, the interpolation polynomial $L\{b,a\}$ can be used to achieve simple and accurate approximation of g^{-1}. In fact, thanks to the interpolation polynomials the estimation of g or g^{-1} corresponds to the same procedure.

A. Polynomial interpolation

We first consider the Lagrange polynomial interpolation, which can be expressed as

$$L_{\text{Lagrange}}\{b,a\}(x) = \sum_{i=1}^{K} a_i \prod_{k \neq i}^{K} \frac{x - b_k}{b_i - b_k}. \tag{16}$$

The second is the Newton form interpolation, which can be expressed as

$$L_{\text{Newton}}\{b,a\}(x) = \sum_{i=1}^{K} c_i \prod_{k=1}^{i} x - b_k, \tag{17}$$

where c_i denotes the *divided difference* as defined iteratively in [14] and is given by

$$c_i = \sum_{k=1}^{i} \frac{a_k}{\prod_{\substack{l=1 \\ l \neq k}}^{i} b_k - b_l}.$$

Note that the obtained polynomials are the same, since there is only one Kth order polynomial which interpolates K+1 points, for the Lagrange and Newton methods.

B. Compensation approach

Compensation for the nonlinearities is now carried out in the time domain using the previously defined inverse interpolation polynomial. The nonlinearity polynomial interpolation function is applied on the amplitude of the received signal, yielding

$$|w(t)| = L\{b,a\}(|y(t)|),$$

where $w(t)$ represents the signal corrected for AM/AM distortion. Subsequently, AM/PM distortion can be evaluated either in the orthonormal basis with (14) or with the interpolation polynomial, yielding

$$\angle \hat{g}(|w(t)|) = \sum_{k=1}^{K} \varphi_k p_k(|w(t)|), \tag{18}$$

where $\angle(\cdot)$ yields the phase of its argument.

Eventually, the received signal is corrected for both the amplitude and phase distortion, yielding

$$w(t) = |w(t)| \exp\left(j\left(\angle y(t) - \angle \hat{g}(|w(t)|)\right)\right). \tag{19}$$

As schematically depicted in Fig. 2, conventional equalization can, subsequently, be applied to the postdistorted signal $w(t)$ to compensate for the wireless channel response.

VI. Simulation Results

To evaluate the performance of the proposed estimation and compensation scheme, simulations were carried out for a 60 GHz OFDM-based system based on the Medium Rate Transmission proposal (MRT1) of the IEEE 802.15.3c group [15]. The relevant system parameters are given in Table 2. This 60 GHz OFDM system is a typical example of a system with high PAPR signals, which puts severe constraints on the power amplification.

Table 2: System simulation parameters.

Number of data sub-carriers	840
Number of sub-carriers	1024
Channel bandwidth	1 GHz
Cyclic prefix duration	96 ns
Modulation	16-QAM
802.15.3c channel	CM1.3, CM3.1, CM2.3
Coding Rate	2/3
PA model	SSPA (p=1.02)
IBO	0 dB

In order to study realistic nonlinearities, we considered a CMOS SOI medium power amplifier (MPA), a picture of whose layout is presented in Fig. 3. The power amplifier achieved 12 dBm output power and a 1dB compression point of 7 dBm. Measurement results of the response of this MPA were matched to those of different amplifier models. It was found that the response of the SSA model with p = 1.02 was very close to that of the implemented MPA. Hence this model was used for simulations.

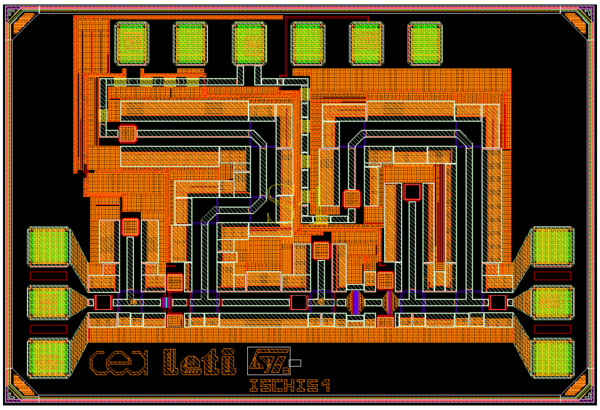

Fig. 3: 60 GHz cascode MPA layout.

Note that, due to the guard subcarriers in the OFDM modulation and the relatively low emitted power of this CMOS amplifier, spectral regrowth remained below the spectrum mask proposed by the IEEE 802.15.3c group.

Due to the 1024 subcarriers and 16-QAM modulation, the OFDM signal exhibits an average PAPR of 8.8 dB. The short training part of the proposed preamble in the MRT1 proposal was replaced by the preamble introduced in Section IV.A, consisting of q_1 and q_2, to allow for estimation of the channel response and nonlinearity, respectively. For q_1 a constant modulus sequence with uniform random phase of one OFDM symbol length is used. For q_2 a random bit sequence modulated OFDM symbol is used, based on the MRT1 data signal. The nonlinear SSA power amplifier model with p = 1.02 was simulated with 0 dB input back off from the 1dB compression point, i.e., a highly nonlinear amplifier operating close to saturation.

For estimation of both the AM/AM and AM/PM characteristics of the nonlinearities, the received training sequence was projected onto a fifth order orthonormal basis, using the method defined in Section II.B. The correction was achieved through a five point Lagrange polynomial interpolation, with a squared linear interpolation basis, as defined in (10).

The accuracy of the inverse (Lagrange) nonlinearity estimation and compensation is studied in Fig. 4. The figure depicts the normalized mean squared error (MSE) between the received linear signal not experiencing nonlinearities $s(t)*h(t)+n(t)$ and the noiseless undistorted signal $s(t)*h(t)$ ("linear"), the MSE between the nonlinear distorted signal $y(t)$ and the noiseless undistorted signal $s(t)*h(t)$ ("distorted"), and the MSE between the compensated signal $w(t)$ and the noiseless undistorted signal $s(t)*h(t)$ ("compensated").

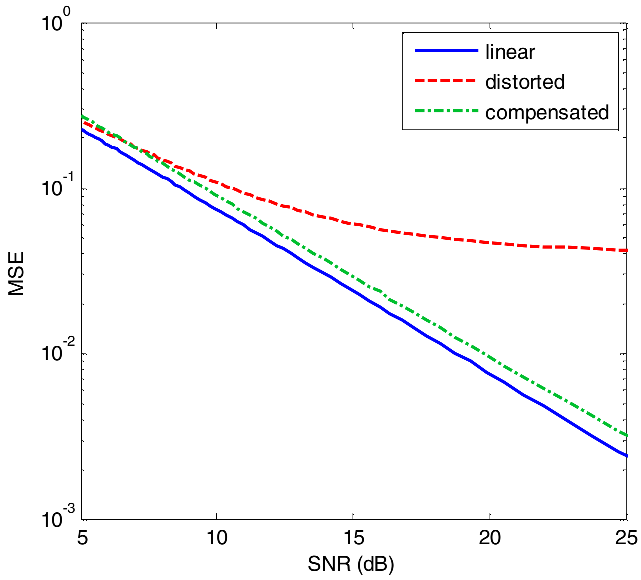

Fig. 4: Normalized mean squared error (MSE) for the distorted and the compensated OFDM system.

The observed MSE indicated by "linear" reveals the influence of the additive noise. Considerable error in the signal is observed for the system experiencing the nonlinearity, indicated by "distorted". When the proposed compensation is applied, shown by the curve indicated by "compensated", only a minor degradation compared the system only experiencing noise is found. Hence, it can be concluded from this figure that the proposed algorithms achieve accurate estimation of the nonlinearity function and its inverse. The main contribution to the error can be attributed to the model mismatch because of limited order of the Lagrange polynomial. The sensitivity to noise can easily be decreased by considering longer training sequences.

Also, Monte Carlo simulations were carried out with CM1.3, CM3.1 and CM2.3 channels with nonlinearity compensation through the Lagrange polynomial interpolation. Figure 5 shows the BER performance for MRT1 and CM3.1 with a squared linear basis for the interpolation points.

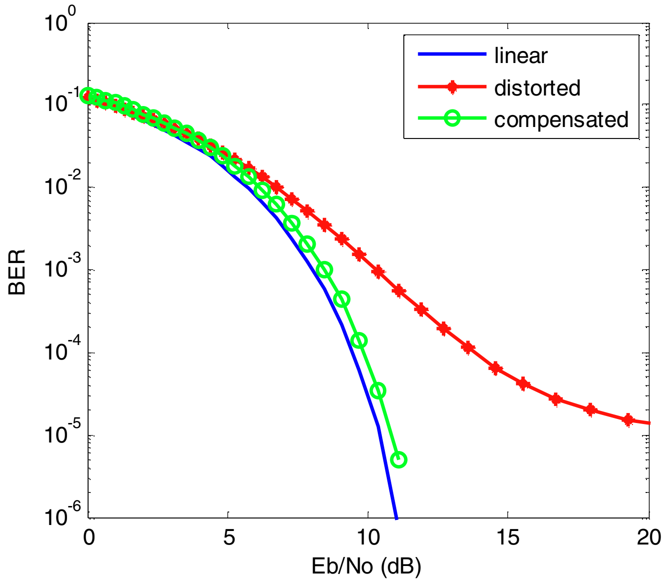

Fig. 5: Bit-error rate (BER) performance for the distorted and the compensated OFDM system for channel CM3.1.

The system without compensation exhibits a significant BER floor in presence of PA nonlinearities for all simulated channels. For SNR values above 5 dB, the estimation and compensation approach successfully reduces the BER to close to the ideal linear signal performance, thus allowing the use of the studied power amplifier up to the 1dB compression point.

VII. Conclusions

A novel approach has been presented to estimate and correct for the in-band amplitude and phase distortion caused by nonlinear power amplifiers in wireless communication systems. The approach is based on postdistortion and polynomial interpolation. Simulation results for a 60 GHz based OFDM system show that the combined estimation and compensation approach can considerably reduce the performance impact of the nonlinearities. As such, the in-band influence of amplifier nonlinearities can be compensated for and will not hinder the application of OFDM in 60 GHz systems. Hence, the achievable PA efficiency will be mainly driven by non-linearity-caused spectral regrowth compared to the imposed emission mask.

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